An Integro-difference Model of Dandelions Spreading

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Abstract: Dandelions have a significant impact on the ecological environment and possess certain economic value. The purpose of this paper is to present an effective mathematical model for predicting the distribution of dandelions. The model is based on the concept of integro-difference equations (IDE), which are particularly effective for modeling the propagation of species with strongly synchronized life stages, influenced heavily by seasonal changes. To utilize the IDE for predicting the spread of dandelions, it is essential to understand the population growth rate of the species. The concept of the logistic model is employed to estimate the population of dandelions. By integrating these mathematical tools, the paper were able to study the spread and growth of dandelions across different seasons and varying conditions, yielding data that forms a normal distribution by distance. As an application, the distribution of dandelions in three different ecological environments is predicted. Additionally, a mathematical model was developed to determine an 'impact factor' for invasive species. This model integrates multiple variables, including the characteristics of the plant and the nature and extent of the harm it inflicts on its environment. The simulation results demonstrate that the model exhibits good predictive performance.

Keywords: Dandelion Distribution; Integro-Difference Equation; Gaussian Distribution; Multivariate Analysis

1. Introduction

1.1 Background

Virtually all human beings have, to varying degrees, blown some sort of parachute-like flower known as dandelions. The excellent experience makes people feel a sense of happiness when they behold the little white umbrella blown into the sky. Dandelions are considered a notable form of flower globally. There are various kinds of dandelions around the world, which attract a lot of people to appreciate them and make a trend of blowing dandelions [1]. As the flower garners more attention, its widespread dispersal has been increasingly observed. This has sparked a heated debate regarding the benefits of dandelions to humans. Dandelions belong to the Aster Family, each flower in this family is a pile of flowers. Commonly found, the name "dandelion" originates from the French word 'dent de lion', referring to the shape of the leaves. They are prevalent throughout North America and Europe [2]. Dandelions, being angiosperms, are capable of producing flowers and seeds. They reproduce asexually through seed production. Typically flowering from March to October, they peak from May to June. Every part of the dandelion is nutritious, with the greens rich in vitamins A, C, and K [3]. Additionally, dandelion roots contain soluble fiber and are known for their potent antioxidants, which may aid in fighting inflammation, controlling blood sugar, and lowering blood pressure. Those discussions online of whether Dandelions are beneficial are based on either single objective factors or subjective experience results. Consequently, a comprehensive evaluation model will be developed to assess factors defining invasive species, which will then be applied to test whether dandelions are invasive [4].

1.2 Problem Restatement

The purpose of this paper is to present an effective mathematical model for predicting the distribution of dandelions. Additionally, a new mathematical model has been developed to identify the core impact factors of plant invasions. The invasion of plants is affected by several factors (eg., moisture, sunlight, soil nutrition, metallic element, the degree of disturbance of the new habitat, new habitat community biodiversity). To get a useful model, we choose some main factors to build our model and neglect some irrelevant factors, which makes our model less complex.

Problem1: A mathematical model needs to be created to predict the spread of dandelions over the course of 1, 2, 3, 6, and 12 months.

Problem2: Formulate a mathematical model capable of determining an 'impact factor' for invasive species. This model should integrate multiple variables, including the plant's characteristics and the nature and extent of the harm it inflicts on its environment.

1.3 List of Variables

Table 1. The Parameter of the Integro-Difference Equations Model

Parameter	Meaning
t	Discrete time
Nt(x)	Spatial density of population
x	Spatial variable x
y	Spatial variable y
Ω	Domain of interest
f	Discrete map
K(x, y)	Dispersal kernel
B	Birth rate
D	Death rate
R_t^i	Population growth rate at time t
i	Growth rates under different climates
σ^2	Variance
δ(x)	Dirac delta distribution function
Nt	Invasive species
Pt	Native species
N⋆	Steady population respectively of invasive species
P⋆	Steady population respectively of native species
v1	Average value of invasive plants
v2	Average value of native plants
ci	The weight of factors
s(x)	Quality of soil affected by invasive plants
w(x)	Quality of water affected by invasive plants

This section outlines the parameters of the Integro-Difference Equations Model (IDE), as detailed in Table 1, which are crucial for our analysis. A clear understanding of these parameters is crucial for grasping how the model simulates the spatial dynamics and distribution of dandelions. The subsequent table outlines each parameter, its function in the model, and the assumptions made during implementation. This detailed breakdown aims to enhance understanding of the model's functionality and ensure transparency in the derivation of simulation results.

1.4 Assumptions

1) It is assumed that each individual dandelion is exactly the same. There are no differences between them in the process of growth and spread. Any variations among individual dandelions are considered negligible, with the probability of occurrence being very small and thus ignored. This assumption facilitates the generalized application of the model.

2) The assumption is made that natural resources in a specific area are finite, thereby limiting the proliferation of dandelions. This assumption is consistent with natural constraints and expectations.

3) The growth rate of dandelions is assumed to be influenced exclusively by climatic conditions, which vary under different climate scenarios. Other potential influencing factors are not considered and therefore remain unanalyzed and uncalculated in this study.

4) The research is set in the United States, thus the assumptions about climate include three typical climate types: temperate, arid, and tropical. It does not consider conditions of extreme cold, extreme heat, or other climates containing extreme factors.

5) The effect of plant density on growth is not considered, and it is assumed that the growth density of dandelions is uniform across a given area.

6) It is assumed that the development and spread of invasive species and native species are identical, and comparisons are made under the same environmental conditions and with the same natural resources. This assumption facilitates the generalization of the models.

2. Model Preparation

The selection of the logistic model for this study was driven by its exceptional adaptability and proven efficacy in handling growth-related issues. The research concentrates on the survival and spread of dandelions within a certain area, a situation where the logistic model excels due to its ability to accurately analyze population dynamics over time. This approach enables an in-depth examination of the factors that influence dandelion proliferation, thereby offering crucial insights into their ecological behavior and potential control strategies [5].

This model is essential when studying population growth because it takes into account the carrying capacity of the environment, meaning it reflects the fact that population growth rate declines as population size (in this case, the number of dandelions) approaches the environment's capacity. By contrast, alternative models such as linear growth models do not adequately consider this important aspect, which limits their effectiveness in accurately prediction in biological scenarios.

Unlike the geometric or exponential growth models, which might initially seem appropriate for a question relating to growth overtime, logistic models are more suitable for our task [6]. While the exponential model assumes an unrestricted growth, and the geometric model considers growth over discrete time periods without taking environmental constraints into account, the logistic model provides a more nuanced view by factoring in both the speed at which the dandelions reproduce, and the constraints imposed by their environment [7]. This is ideal for analyzing plant species like dandelions which reproduce rapidly but are also impacted by factors like nutrient availability, sunlight, and competition with other species. Furthermore, the logistic model can neatly cater to pace changes in the population of dandelions. In the early stages of growth when competition for resources is limited, this model allows for a rapid increase in population - similar to the exponential model. However, as the population nears its saturation point, the logistic model demonstrates a slow growth pattern which other models fail to showcase [8].

In conclusion, the degree of uncertainty is reduced when using logistic models because they account for a variety of influencing factors, enhancing prediction accuracy for resource planning and management in the study area. Consequently, given the biological nature and specific constraints of the study, a logistic model emerges as the ideal mathematical framework for investigating the dynamics of the dandelion population in the area.

3. Results Analysis

3.1 Problem 1: The Spread of Dandelions

3.1.1 Basic Info

Dispersal is an important aspect of population dynamics and biology. There are many mathematical models studying the spread or invasion of some species in a region. The simplest method of modeling dispersal is to ignore specific spatial interactions or as simple transfer functions. Integro-difference equations are a popular tool used in theoretical ecology to model spreading populations [9]. The main ideal of integro-difference equations is to view the spreading behavior as two stages: the growth phase and dispersal phase. At the beginning of a season, seeds grow and develop leaves, flowers and seeds. The plants do not move in this phase. Later in the season, plants release seeds that travel to a different location, usually by wind or birds. Note that in this model, the spreading process is separated to a growth phase and a dispersal phase with no overlapping.

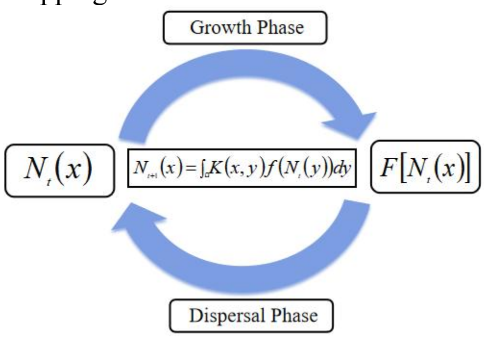

Figure 1. The Integro-Difference Equations Model.

To develop a mathematical model for the spreading process depicted in Figure 1, the spatial density of the population at discrete time t is denoted as Nt(x). The spatial variable x exists within a specific domain of interest, represented by Ω. The growth phase is modeled using a discrete map f. During the dispersal phase, individuals move from one location to another based on certain probabilities. The outcome of this phase is captured by a dispersal kernel K(x, y), which quantifies the probability of an individual traveling from location y to location x. Consequently, the integro-difference equation representing this process is expressed as follows:

$$N_{t+1}(x)=\int_{\Omega} K(x, y)f(N_t(y))dy. \quad (1)$$

There are many types of growth functions which describe the various ecological mechanisms, such as Malthus model and logistic model [10]. The Malthus model is the simplest model to describe the growth of a single population. It is based on some simple population assumptions:

1) The individuals are identical.
2) There is no immigration.
3) The population is very large, allowing for the assumption that the population function is differentiable.

The spatial variable x is disregarded, allowing N_t to serve as the population function dependent on the time variable t. The dynamics of the population are driven uniquely by the processes of birth and death. The birth rate and death rate are denoted by B and D respectively. Assuming that both rates are constant, this leads to the adoption of the Malthus model:

$$\frac{dN_t}{dt} = B - D. \quad (2)$$

The Malthus model concludes that the population growths exponentially, which is not true during a long period of time. An increase of the population size produces a fertility decrease and a mortality increase, since resources are limited. This is the so called logistic effect. Assuming that fertility declines and mortality increases are linear functions of the population, the resultant framework leads to the adoption of the logistic model:

$$N_t^{'} = r\left(1 - \frac{N_t}{N_{max}}\right)N_t \quad (3)$$

3.1.2 The Model of Dandelions Spreading

The task is to construct a mathematical model capable of predicting the spread of dandelions over 1, 2, 3, 6, and 12 months. This model is developed using the principles of integro-difference equations. Given that one year is not considered a lengthy period, the Malthus model is employed to forecast population growth. Specifically,

$$f\left(N_t(x)\right) = R_t^i \cdot N_t(x). \quad (4)$$

where R_t^i denotes the population growth rate at time t. To predict the dispersal of dandelions over an extended period, the Malthus model can be replaced with the logistic model. It is important to note that the growth rate of dandelions varies with changes in climate. The index i is used to distinguish the growth rates under different climatic conditions.

The seeds of dandelions are easy to spread by wind. Initially, a dandelion is located adjacent to an open one-hectare plot of land, as depicted in Figure 2. It is assumed that the primary factor influencing the spread of dandelions is the wind, and the probability of a seed traveling from location x to location y depends solely on the distance $\|x - y\|$. Empirically, the probability of a long travel is smaller than a short distance travel. Therefore, it is assumed that the dispersal kernel function K(x, y) follows a normal distribution, commonly referred to as the Gaussian kernel:

$$K(x, y) = G(x - y) = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{\|x-y\|^2}{2\sigma^2}\right) \quad (5)$$

where σ^2 is the variance, and x, y $\in R^2$.

The puffball of a dandelion consists of numerous seeds, each capable of germinating into new plants. It is assumed that an initial puffball contains 100 dandelion seeds, leading to the initial distribution function $N_0(x) = 100\delta(x)$, where δ(x) represents the Dirac delta distribution [11]. Combining this information, the following is derived:

$$N_{t+1}(x) = \int_{R^2} G(x - y)f(N_t(y))dy \quad (6)$$

Note that the above integral is just the convolution of functions G(x) and $f(N_t(x))$. It is known that $f = (f * \delta)(x)$ for any continuous function f. Utilizing the properties of convolution and the initial condition, the following results are obtained:

$$N_t(x) = 100\prod_{k=1}^{t} R_k^i G^{*t} \quad (7)$$

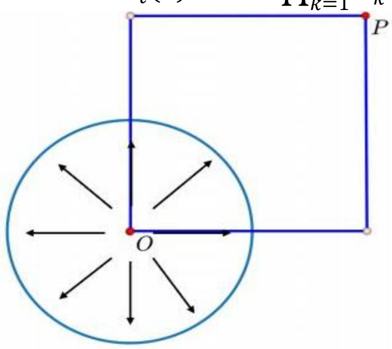

Figure 2. A diagram of the Spread of Dandelions.

The t-fold convolution of a Gaussian kernel with zero mean and variance σ^2 is a Gaussian kernel with zero mean and variance $t\sigma^2$. The above formula can be expressed as follows, serving as the prediction model.

$$N_t(x) = 100\prod_{k=1}^{t} R_k^i \frac{1}{2t\sigma^2\pi}\exp\left(-\frac{\|x\|^2}{2t\sigma^2}\right) \quad (8)$$

3.1.3 Simulations and Applications

1) Case one: temperate climate

The data on the population growth of dandelions in a temperate climate over one year is presented in Table 2. Meanwhile, the following Figures 3, 4, and 5 depict the distribution diagrams of dandelions over several months.

Table 2. The Population Growth of Dandelion Intemperate Climate for One Year

t/month	1	2	3	4	5	6	7	8	9	10	11	12
R_t^1	0.5	0.5	4	4	4	2	2	2	1.5	1.5	1.5	0.5

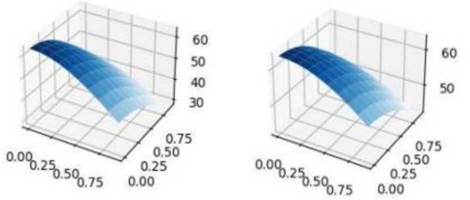

Figure 3. The Left Image Shows the Population of a Hectare of Dandelions in the First Month in Temperate Climates, While the Right Image Depicts the Population in the Second Month.

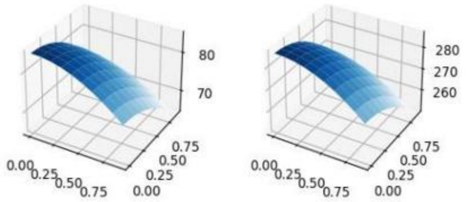

Figure 4. The Left Image Displays the Population of a Hectare Of Dandelions in the Third Month in Temperate Climates, Whereas the Right Image Depicts the Population in the Sixth Month.

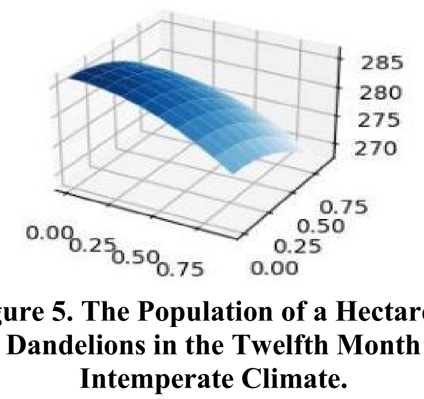

Figure 5. The Population of a Hectare of Dandelions in the Twelfth Month Intemperate Climate.

2) Case two: arid climate

In arid climates, the mortality rate of dandelions significantly increases. Data that monitor their population growth over one year vividly demonstrate this decline, as detailed in Table 3. Additionally, Figures 6, 7, and 8 effectively illustrate the temporal distribution patterns of dandelions across several months, capturing the impact of these harsh conditions.

Table 3. The Population Growth of Dandelion in Arid Climate for One Year

t/month	1	2	3	4	5	6		7	8	9	10	11	12
R_t^2	0.4	0.4	3	3	3	1.5		1.5	1.5	2	2	2	0.4

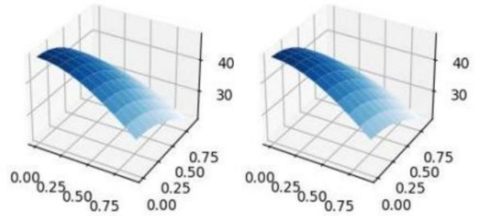

Figure 6. The Left Image Shows the Population of a Hectare of Dandelions in the First Month in Arid Climates, While the Right Image Depicts the Population in the Second Month.

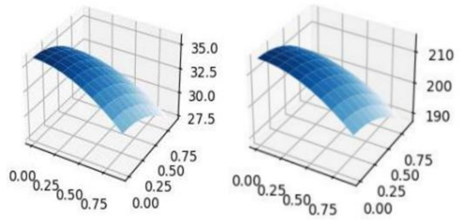

Figure 7. The Left Image Displays the Population of a Hectare of Dandelions in the Third Month in Arid Climates, Whereas the Right Image Depicts the Population in the Sixth Month.

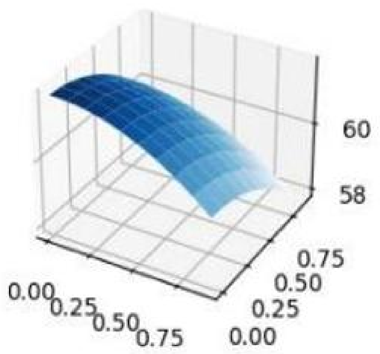

Figure 8. The Population of a Hectare of Dandelions in the Twelfth Month in Arid Climates.

3) Case three: tropical climate.

In a tropical climate, the distinction between the four seasons is less pronounced, replaced instead by marked dry and rainy periods. Data tracking the population growth of dandelions over one year clearly highlight these seasonal variations, as shown in Table 4. Furthermore, Figures 9, 10, and 11 effectively illustrate the temporal distribution patterns of dandelions over several months, capturing the effects of these seasonal changes.

Table 4. The Population Growth of Dandelion in Tropical Climate for One Year

t/month	1	2	3	4	5	6	7	8	9	10	11	12
R_t^3	0.6	0.6	0.6	5	5	5	0.8	0.8	0.8	4	4	4

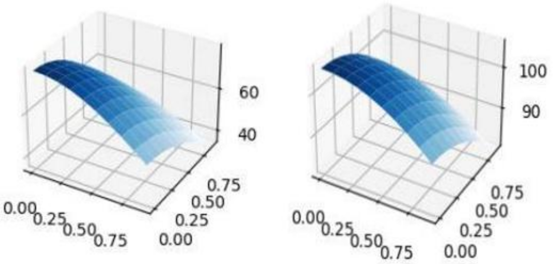

Figure 9. The Left Image Shows the Population of a Hectare of Dandelions in the First Month in Tropical Climates, While the Right Image Depicts the Population in the Second Month.

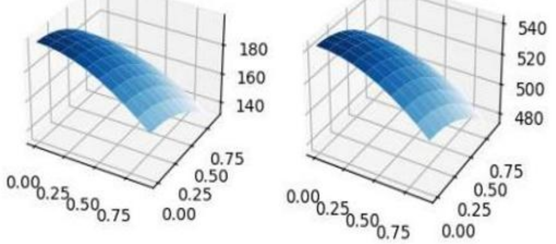

Figure 10. The Left Image Displays the Population of a Hectare of Dandelions in the Third Month in Tropical Climates, Whereas the Right Image Depicts the Population in the Sixth Month.

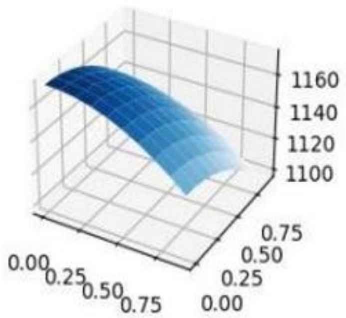

Figure 11. The Population of a Hectare of Dandelions on the Twelfth Month in Tropical Climates.

Across these three scenarios, it becomes evident that increased distance presents greater challenges for dispersal, while favorable climates significantly enhance the reproductive rate of dandelions.

3.2 Problem 2: The Impact Factor for Invasive Species

3.2.1 A General Model on Invasive and Native Species

In problem 1, it only considers the population dynamics of a single species. However, in reality, different species in the same space interact with each other. These interactions can be beneficial or detrimental, depending on the relationships of the involved species. Thus, to simplify the discussion, it is assumed that there are only two species involved, namely the invasive species and the native species. Consequently, since the invasive species impacts the initial environment, the following general model is derived:

$$N_{t+1} = F(N_t, P_t),\ P_{t+1} = H(N_t, P_t) \quad (9)$$

There, the time is discrete, and symbols are used to denote the invasive species and native species respectively. The interactions can be roughly divided into three categories, depending on how one species affects the growth function of the other.

1) Prediction of N by P if $\frac{\partial F}{\partial P} < 0$ and $\frac{\partial H}{\partial N} > 0$.

2) There is competition between N and P if $\frac{\partial F}{\partial P} < 0$ and $\frac{\partial H}{\partial N} > 0$.

3) Mutual facilitation between N and P if $\frac{\partial F}{\partial P} > 0$ and $\frac{\partial H}{\partial N} > 0$.

After a long period of time, the population of invasive species and native species go to steady. The Symbols N^* and P^* are used to denote the steady populations respectively. These satisfy the following equations:

$$N^* = F(N^*, P^*),\ P^* = H(N^*, P^*) \quad (10)$$

The fixed point (N^*, P^*) is stable, allowing for the investigation of its local stability through the eigenvalues of the Jacobian matrix [12].

$$J = \begin{bmatrix} \frac{\partial F}{\partial N} & \frac{\partial F}{\partial P} \\ \frac{\partial H}{\partial N} & \frac{\partial H}{\partial P} \end{bmatrix} \quad (11)$$

It is known that a fixed point is local stable if both eigenvalues are inside the unit circle, which is equivalent to the following conditions.

$$1 - \text{tr } J + \det J > 0, \quad (12)$$

$$1 + \text{tr } J + \det J > 0. \quad (13)$$

$$1 - \det J > 0. \quad (14)$$

3.2.2 The Model of Impact Factors

To develop a model on the impact factors of invasive species, it is necessary to consider its impacts on the environment. Each plant holds some value, so the model should include variables for the final populations of each species. This leads to a modification of the model from the previous paragraph to predict (Nt, Pt).Since there is a competitive relation between different plants, the model belongs to the second case, i.e. $\frac{\partial F}{\partial P}$<0 and $\frac{\partial H}{\partial N}$<0.

Plants exhibit a competitive relationship primarily because they compete for soil and water. Consequently, it can be assumed that the competitiveness of plants is directly proportional to their quantity, making the growth functions F and H linear. The model is thus written as follows:

$$\begin{bmatrix} N_{t+1} \\ P_{t+1} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_t \begin{bmatrix} N_t \\ P_t \end{bmatrix} \quad (15)$$

where aij depend on the characteristic of each plant. Note that different plants have different competitiveness in different environments. The numbers aij vary as time change.If the numbers aij remain constant during a period of time, one can directly check whether a fixed point is locally stable, by using the conditions mentioned in the previous paragraph. To simplify the notation,At = (aij)t is written. With a given initial value, the population at time t can be computed directly:

$$\begin{bmatrix} N_t \\ p_t \end{bmatrix} = \prod_{k=1}^{t} A_k \begin{bmatrix} N_0 \\ p_0 \end{bmatrix} \quad (16)$$

In addition to affecting other plants, invasive plants can also impact local soil and water quality. By combining these factors, one arrives at the model of impact factor at time t:

$$I_t = c_1\left(\frac{v_1 N_t + v_2 P_t}{v_2 P_0} - 1\right) + c_2 s(N_t) + c_3 w(N_t) \quad (17)$$

where v1 and v2 denote the average value of invasive plants and native plants respectively, the numbers ci represent the weight of these factors. The functions s(x) and w(x) denote quality of soil and water affected by invasive plants, respectively.

If the populations of invasive and native plants stabilize after a sufficient period, the final impact factor can then be computed as follows.

$$I^* = c_1\left(\frac{v_1 N^* + v_2 P^*}{v_2 P_0} - 1\right) + c_2 s(N^*) + c_3 w(N^*) \quad (18)$$

The numbers ci, vi and the value of s(x) and w(x) can be determined by experience.

3.2.3 Simulations ad Applications

1) Case one: coexistence

Dandelion is a widely distributed invasive plant whose seeds fly and are able to survive in a variety of environments. Although dandelion invasion causes competitive pressure on existing plants, in some cases they are also able to coexist with other plants. Dandelions are highly adaptable to the soil, helping to improve soil structure, and dandelion roots are relatively shallow, but they can quickly expand and occupy large areas. This causes water and nutrients from the soil to be absorbed by the dandelions, putting competitive pressure on other plants and affecting their growth and development. This shallow root system improves soil permeability. Therefore, in some ecosystems, dandelions form a relatively balanced coexistence with other plants.

It is assumed that dandelions invade a specific area in March when the climate conditions are especially favorable for their growth. As spring concludes, these dandelions begin to compete with the local plants for survival. The following data, presented in Table 5, shows the coefficients a(i, j) for case one across different months.

Table 5. Coefficients A(i, j) for Case one in Different Month.

t/month	1	2	3	4	5	6	7	8	9	10	11	12
a11	0.9	0.9	4	4	4	1.3	1.3	1.3	1.3	1.3	1.3	0.9
a12	-0.1	-0.1	-0.2	-0.2	-0.2	-0.5	-0.5	-0.5	-0.2	-0.2	-0.2	-0.1
a21	-0.1	-0.1	-2	-2	-2	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3	-0.1
a22	0.9	0.9	1.5	1.5	1.5	1.5	1.5	1.5	1.3	1.3	1.3	0.9

The changes in the quantities of dandelions and local plants can be determined. Presented in Figure 12 are the annual population trends of dandelions and native plants.

Dandelion has certain medicinal value and can improve soil. On the other hand, dandelions consume more water resources. The coefficient of c1, c2, and c3 are set to 1, with s(N11) established at 1 and w(N11) marked at −1. Consequently, if the value of dandelion and local plants are similar, then formulation (19) is derived.

$$I_{11} = \left(\frac{116+115}{100} - 1\right) + 1 - 1 = 1.31 \quad (19)$$

2) Case two: destruction of original plants

Purple Loosestrife is an invasive plant from Europe that is known for its showy purple flowers. However, the invasion of Purple Loosestrife has a devastating effect on wetland ecosystems because it has a strong growth capacity and can form dense communities that gradually crowd out the original wetland plants. Purple Loosestrife can influence soil quality by changing nutrient cycling patterns. The plant's decaying organic matter may contribute to an increase in nutrient levels, altering the composition of the soil and potentially favoring the growth of other invasive species. This invasive plant not only reduces wetland biodiversity, but can also lead to a decline in soil quality, negatively impacting the habitat of birds and other wetland animals. In April, Loosestrife invaded a region, proving more competitive than the local plants. Table 6 below shows the coefficients a(i, j) for different months under scenario two. And Figure 13 illustrates the annual changes in the populations of Loosestrife and local plants as follows.

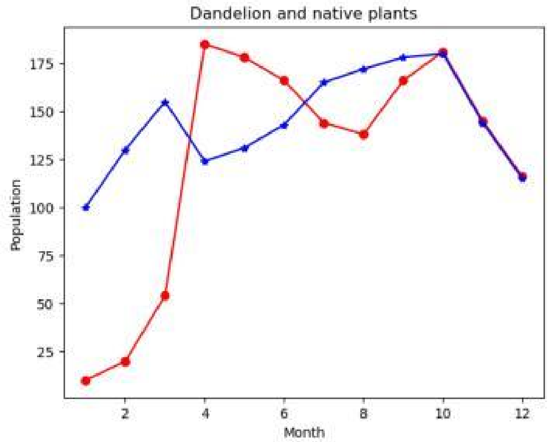

Figure 12. The Population of Dandelion and Native Plants in One Year (The Red Line Represents the Invasive Plants; the Blue Line Represents the Native Plants).

Table 6. Coefficients A(i, j) for Case Two in Different Month.

t/month	1	2	3	4	5	6	7	8	9	10	11	12
a11	0.8	0.8	3	3	3	2	2	2	1	1	1	0.8
a12	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1
a21	0	0	-2	-2	-2	-1	-1	-1	-0.3	-0.3	-0.3	0
a22	0.8	0.8	1.5	1.5	1.5	1.5	1.5	1.5	1.2	1.2	1.2	0.8

Although loosestrife has certain medicinal value, the quality of soil and water is more important for wetlands. Consequently, the coefficient c1 is set to 1, while c2 and c3 are both established at 2. Due to loosestrife's detrimental effects on soil and water quality, s(N11) and w(N11) are each assigned a value of −1. It is further presumed that the value of native plants parallels that of loosestrife. Proceeding from these assumptions, the subsequent results are derived:

$$I_{11} = \frac{91}{100} - 1 + 2(-1) + 2(-1) =- 4.09 \quad (20)$$

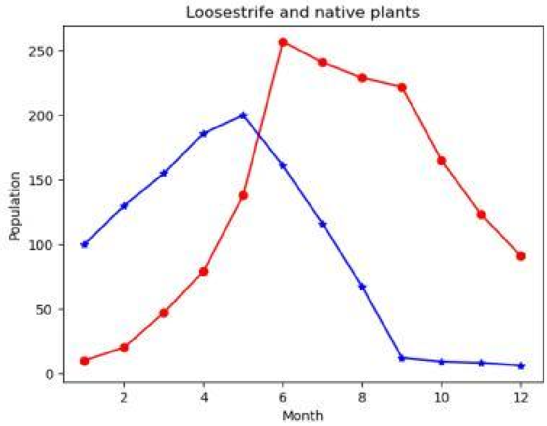

Figure 13. The Population of Loosestrife and Native Plants in One Year (The Red Line Represents the Invasive Plants; the Blue Line Represents the Native Plants).

3) Case three: destroyed

Honey Locust is a plant native to North America, but has become an invasive species in some areas. Its invasion has multiple effects on the environment, soil and other plants. Honey Locust invasion can change the structure and dynamics of local ecosystems. Due to the strong growth ability of Honey Locust, it may form a large area of single vegetation type, replace the original ecological niche, and have a negative impact on ecological diversity. Moreover, the root system of Honey Locust is deep and strong, sometimes forming a strong network of roots. This leads to problems with soil erosion, as roots stabilize the soil surface, but at the same time may limit the growth of other plants and affect the overall health of the soil. Honey Locust also prefers wet, watery environments, which can lead to water scarcity. However, due to the extremely slow breeding of Honey Locust, the invasion of acacia is very easy to stop artificially. Figure 14 illustrates the population changes of Honey Locust and native plants over the course of one year.

Honey Locust invaded a certain area in March and has strong competitiveness, causing damage to local plants, but its growth is slow. Table 7 displays the coefficients a(i, j) for case three across different months.

Table 7. Coefficients a(i, j) for Case Three in Different Month.

t/month	1	2	3	4	5	6	7	8	9	10	11	12
a11	0.9	1	1	1	1	1	1	1	1.1	1.1	1.1	0.9
a12	0	0	0	0	0	0	0	0	0	0	0	0
a21	-2	-2	-4	-4	-4	-2	-2	-2	-3	-3	-3	-2
a22	0.8	0.8	1.4	1.4	1.4	1.1	1.1	1.1	1.2	1.2	1.2	0.8

Due to the toxicity of Honey Locust, the coefficient v1 is set to −2 and v2 to 1. This species damages the soil but has minimal impact on water quality, prompting the assumptions that s(N11) = 1, w(N11) = 0, and c1, c2, and c3 are all set to 1. Consequently, the following formulation is established:

$$I_{11} = \left(\frac{-11+17}{100} - 1\right) - 1 + 0 = -1.93 \quad (21)$$

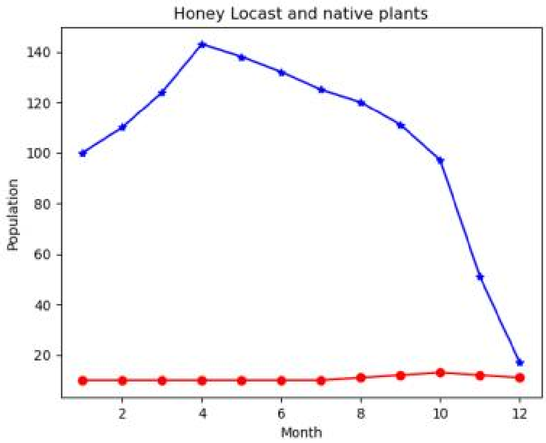

Figure 14. The Population of Honey Locust and Native Plants in One Year (the Red Line Represents the Invasive Plants; the Blue Line Represents the Native Plants)

4. Conclusion

In this study, the paper is to present an effective mathematical model for predicting the distribution of dandelions. By building the integro-difference equations (IDE) model, the mechanism of the distribution of dandelions is discussed through correlation analysis. Additionally, a mathematical model was developed to determine an ‘impact factor’ for invasive species. The model analyzes and discusses the nature and extent of the environmental harm caused by factors such as the characteristics of plants through multivariate analysis methods.

The calculation demonstrates that the first model exhibits robust predictive performance when supplied with accurate data. Importantly, this model boasts a significant advantage due to its simplicity of computation. Additionally, it incorporates distance, enhancing both its realism and reliability in real-world scenarios. The inclusion of distance enables the model to more effectively capture local spread characteristics, where proximity plays a crucial role in enhancing propagation. Such local consideration potentially improves the model's accuracy in specific contexts. However, the first model encounters challenges in modeling low-probability events and may struggle with accurate predictions in scenarios where seeds spread to distant locations.

The second model is capable of effectively estimating the impact factors of dandelion on different ecological environments. The strength of this model lies in its intuitiveness, where larger numerical values indicate more favorable conditions for invasive plants, while smaller values signify greater harm. This representation is straightforward, enhancing the model's interpretability and allowing for a clear explanation of the impact of invasive plants on the environment. However, in some special cases, the accuracy of the second model's estimates may falter. Constrained by distance factors, the model might struggle to accurately capture phenomena such as the jumping spread of seeds, potentially leading to inaccuracies in predictions under specific circumstances. Furthermore, a notable drawback of the second model is the challenge in obtaining certain coefficients, which often require reliance on empirical data or experimental results. This dependency can make the establishment and adjustment of the model relatively difficult, particularly in the absence of prior knowledge.

In summary, the model proposed in this paper exhibits good predictive performance under general conditions. Furthermore, the future work will explore performance under extreme conditions and additional influencing factors to further optimize and expand the model.

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