Academic Education
 **Kinetic Analysis of Fractional Viscoelastic Films Based on

Shifting Legendre Polynomials

Yunchen Liu^{1,2}, Yanbing Liang^{2,*}
** *Rebei Key Laboratory of Data Science and Application, North China Uni Industry Science and Engineering Vol. 1 No. 10, 2024*
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Industry Science and Engineering Vol. 1 No. 10, 20
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Yunchen Liu^{1,2}, Yanbing Liang^{2,*}

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 2College of Science, North China University of Science, North China University of Science and Technology, Tangshan, Hebei,} *Industry Science and Engineering Vol. 1 No. 1*
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Shifting Legendre Polynomial:

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1. Introduction

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PET film is widely used in packaging,

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PET film is widely used in packaging,

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membrane and other industries, showing its

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PET film is generally produced by roller-to-

roller manufacturing, which is an advanced

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Viscoelastic Films Based on

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 cce and Technology, Tangshan, Hebei, China
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Many scholars have studied the film: Jimei Wu

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influence of film width, amplitude and
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Most of the current studies are based on
integer-order and fractional-order models to
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Most of the current studies are based on

integer-order and fractional-order models to

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integer-order and fractional-order models to
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between the actua integer-order and fractional-order models to
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experimental results [5] However, for a
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phenomena, which cannot eliminate the error
between the actual material properties and the
experimental results [5] However, for a
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highly l phenomena, which cannot eliminate the error
between the actual material properties and the
experimental results [5] However, for a
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highly likely that large deformation will o between the actual material properties and the experimental results [5] However, for a composite flexible substrate such as PET, it is highly likely that large deformation will occur during the production process [6]. Ther experimental results [5] However, tor a
composite flexible substrate such as PET, it is
highly likely that large deformation will occur
during the production process [6]. Therefore, a
model that takes into account the
visc

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materials compared with traditional models

have emerged. In this paper, we are committed

to the dynamic analysis of thin films on rollers

using a variational frac **Industry Science and Engineering Vol. 1 No. 10, 2024**

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using a variational fractional order model.

The structure of this materials compared with traditional mo
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using a variational fractional order model.
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 2. The structure of this paper is as follows:
 2. Establishes the partial differential
 2. Establishment of Nonlinear Differential
 2. Establishment of Nonlinear Diff using a variational riactional order model.

The structure of this paper is as follows:

Section 2 establishes the partial differential

governing equations for variable fraction films.

Section 3 converts the governing e Section 2 establishes the partial differential

governing equations for variable fraction films.

Section 3 converts the governing equations of

thin films into matrix form based on the

differential operation matrix of t governing equations for variable fraction films.

Section 3 converts the governing equations of

thin films into matrix of the shifted

Legendre polynomial. Section 4 gives error

analysis and mathematical examples of
 $\$ Section 3 converts the governing equations of

thin films into matrix form based on the

differential operation matrix of the shifted

Legendre polynomial. Section 4 gives error

analysis and mathematical examples of

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Microbeams

thin tilms into matrix form based on the

differential operation matrix of the shifted

Legendre polynomial. Section 4 gives error

analysis and mathematical examples of $(\frac{\partial^2 w}{\partial t^2} + 2c \frac{\partial^2 w}{\partial \bar{x} \partial \tau} + c^2 \frac{\partial^2 w}{\$ dulterential operation matrix of the shitted

Legendre polynomial. Section 4 gives error

analysis and mathematical examples of $(\frac{\partial^2 w}{\partial \tau^2} + 2c \frac{\partial^2 w}{\partial \bar{x}} + c^2 \frac{\partial^2 w}{\partial \bar{x}^2})$

algorithms. Section 5 gives the con Legendre polynomial. Section 4 gives error $(\frac{\partial^2 w}{\partial t^2} + 2c \frac{\partial^2 w}{\partial x \partial t} + c \frac{\partial^2 w}{\partial x \partial t})$

algorithms. Section 5 gives the conclusion of $+$

this paper.
 2. Establishment of Nonlinear Differential Governing Equat analysis and mathematical examples of ∂x^2
algorithms. Section 5 gives the conclusion of $+\beta y \frac{\partial^2 w}{\partial x^2}$
be satisfied.
2. Extablishment of Nonlinear Differential
Governing Equations for Viscoelastic $\begin{cases} w(0$ algorithms. Section 5 gives the conclusion of $f(\theta) = \theta$

this paper.

2. **Establishment of Nonlinear Differential**
 Governing Equations for Viscoelastic

Microbeams

In the manufacturing process of the film, it is

the this paper.
 Soundary conditions can be 2
 Extablishment of Nonlinear Differential
 Governing Equations for Viscoelastic ${w(0, y, \tau) = w(1, y, \tau) = 0 \over \frac{\partial^2 w(0, y, \tau)}{\partial x^2}$

In the manufacturing process of the film, it 2. Establishment of Nonlinear Differential

Governing Equations for Viscoelastic

Microbeams

In the manufacturing process of the film, it is

The initial condition

assumed that the film is uniformly continuous,

follows 2. Establishment of Nonlinear Differential

Governing Equations for Viscoelastic

Microbeams

In the manufacturing process of the film, it is

The initial condition can be

assumed that the film is uniformly continuous,
 Governing Equations for Viscoelastic

Microbeams

In the manufacturing process of the film, it is

assumed that the film is uniformly continuous,

follows elasticity and isotropy, and is not

affected by bending stiffne **Microbeams**

In the manufacturing process of the film, it is

assumed that the film is uniformly continuous,

follows elasticity and isotropy, and is not

affected by bending stiffness and shear forces

due to its light In the manufacturing process of the film, it is

saysumed that the film is uniformly continuous,

follows elasticity and isotropy, and is not

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due to its light weight and so assumed that the film is uniformly continuous,

follows elasticity and isotoropy, and is not

affected by bending stiffness and shear forces

due to its light weight and soft properties [7] in

addition, the large deforma follows elasticity and isotropy, and is not
affected by bending stiffness and shear forces
due to its light weight and soft properties [7] in
addition, the large deformation assumption
should be satisfied. That is, compar mgular film under external

brane has a length of *L*and

inckness of *h*, extern
 $\therefore \cos{(wt)}$, and the film
 $\therefore \cos{(wt)}$, and the film
 $\therefore \sin{(wt)}$. The governing equat

can be formulated as:
 $\frac{w}{z} + 2v \frac{\partial^2 w}{\partial x \partial t} + v^$ r film under external excitations

a length of Land a width

ess of h, external inc

(wt), and the film is delivery

The governing equations of the formulated as:
 $\frac{\partial^2 w}{\partial x \partial t} + v^2 \frac{\partial^2 w}{\partial x^2} - N_x \frac{\partial^2 w}{\partial x^2} - N_y \frac{\$ nder external excitation. The

ength of Land a width of band

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nd the film is delivered at

verning equations of the thin

ated as:
 $\frac{\partial^2 w}{\partial x^2} - N_y \frac{\partial^2 w}{\partial y^2}$
 $T_0(1 + \beta \frac{y}{b}) \frac{\partial^2 w}{\partial x^2}$ 2 − $\int f h$, external incentives

and the film is delivered at

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ulated as:
 $\int_2^2 \frac{\partial^2 \bar{w}}{\partial x^2} - N_x \frac{\partial^2 \bar{w}}{\partial x^2} - N_y \frac{\partial^2 \bar{w}}{\partial y^2}$
 $- T_0 (1 + \beta \frac{y}{b}) \frac{\partial^2 \bar{w}}{\partial x^2} - F \cos(\omega t)$

= 0

m 2
 $\frac{2}{3}$ $\frac{2}{x^2} - N_y \frac{\partial^2 w}{\partial y^2}$
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 $\frac{2}{3}$ $\frac{2}{x^2} - F \cos(\omega t)$
 $\frac{2}{3}$ $\frac{2}{x^2} - F \cos(\omega t)$
 $\frac{2}{x^2} - F \cos(\omega t)$
 original size, the derormation of the film under
stress cannot be ignored. Figure 1 is a
schematic representation of a viscolatic
rectangular film under external excitation. The
membrane has a length of Land a width of ba stress cannot be ignored. Figure 1 is a

schematic recetangular film under external excitation. The

membrane has a length of Land a width of band

a thickness of h, external incentives at Simple Hari

for F. cos (wt), an rectangular film under external excitation. Ine

an thickness a length of Land a width of band

for *F. cos* (wt), and the film is delivered at

velocity v. The governing equations of the thin

film can be formulated as:

For *F*. cos (*wt*), and the film is delivered at
velocity *v*. The governing equations of the thin
film can be formulated as: **Legendre**

$$
\rho \left(\frac{\partial^2 w}{\partial t^2} + 2v \frac{\partial^2 w}{\partial x \partial t} + v^2 \frac{\partial^2 w}{\partial x^2} \right) - N_x \frac{\partial^2 w}{\partial x^2} - N_y \frac{\partial^2 w}{\partial y^2}
$$
 Define

$$
- T_0 (1 + \beta \frac{y}{b}) \frac{\partial^2 w}{\partial x^2} - F \cos (\varpi t)
$$
 (1)

$$
= 0
$$
 where ρ is the density of the membrane. T_x is
the variable axial tension, *w* indicates lateral
displacement.
The following dimensionless quantities are
interduced: \Box

displacement.

where
$$
\rho
$$
 is the density of the membrane. T_x is
\nthe variable axial tension, *w* indicates lateral
\ndisplacement.
\nThe following dimensionless quantities are
\nintroduced:
\n
$$
w = \frac{\overline{w}}{h}, \overline{x} = \frac{x}{L}, \overline{y} = \frac{y}{b}, \tau = t \sqrt{\frac{Eh^3}{\rho L^4}}, c = v \sqrt{\frac{\rho L^2}{Eh^3}}, \zeta
$$
\n
$$
= \frac{1}{2(1 - \mu^2)}
$$
\n
$$
r = \frac{L}{b}, F = \overline{F} \frac{L^4}{Eh^4}, \eta = \overline{\eta} \left(\frac{Eh^3}{\rho L^4} \right)^{\alpha(t)}, \omega
$$
\n
$$
= \frac{\overline{w}}{\sqrt{\frac{\rho L^4}{Eh^3}}}
$$
\nwhere $Z(x) = [1, x]$
\n
$$
r = \frac{L}{b}, F = \overline{F} \frac{L^4}{Eh^4}, \eta = \overline{\eta} \left(\frac{Eh^3}{\rho L^4} \right)^{\alpha(t)}, \omega
$$
\nwhere $Z(x) = [1, x]$
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= \frac{a_{0,0}}{a_{1,0}}
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= \frac{a_{1,0}}{a_{1,0}}
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= \frac{a_{0,0}}{a_{1,0}}
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\n
$$
= \frac{a_{1,0}}{a_{1,0}}
$$
\n
$$
= \frac{a_{0,0}}{a_{1,0}}
$$
\n
$$
= \frac
$$

The above equation becomes:
\n
$$
-\zeta(1 + \eta D^{\alpha(t)}) \left[\left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial x^2} + \mu r^2 \left(\frac{\partial w}{\partial y} \right)^2 \frac{\partial^2 w}{\partial x^2} + r^4 \left(\frac{\partial w}{\partial y} \right)^2 \frac{\partial^2 w}{\partial y^2} + r^4 \left(\frac{\partial w}{\partial y} \right)^2 \frac{\partial^2 w}{\partial y^2} + \mu r^2 \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial y^2} \right]
$$
\n
$$
= \left(\frac{\partial^2 w}{\partial \tau^2} + 2c \frac{\partial^2 w}{\partial x \partial \tau} + c^2 \frac{\partial^2 w}{\partial x^2} \right) - T_0 (1 + \beta y) \frac{\partial^2 w}{\partial x^2} - F \cos \omega \tau = 0
$$
\nBoundary conditions can be expressed as:
\n
$$
\left(\frac{w(0, y, \tau)}{w(x, 0, \tau)} = \frac{w(1, y, \tau)}{w(x, 0, \tau)} = 0 \right) \frac{\frac{\partial^2 w(0, y, \tau)}{\partial x^2}}{\frac{\partial^2 w(0, y, \tau)}{\partial y^2}} = \frac{\partial^2 w(0, y, \tau)}{\partial y^2} = 0
$$
\nThe initial condition can be expressed as [8]:
\n
$$
\left(\frac{w(\bar{x}, \bar{y}, \tau)}{w(\bar{x}, 0, \tau)} \Big|_{\tau=0} = 0
$$
\n
$$
\left\{ \frac{\partial w(\bar{x}, \bar{y}, \tau)}{\partial \tau} \Big|_{\tau=0} = 0 \right\}
$$

$$
\begin{aligned}\n\{\mathbf{w}(0,\bar{\mathbf{y}},\tau) = \mathbf{w}(1,\bar{\mathbf{y}},\tau) = 0 \quad \left| \frac{\partial^2 \mathbf{w}(0,\bar{\mathbf{y}},\tau)}{\partial x^2} = \frac{\partial^2 \mathbf{w}(L,\bar{\mathbf{y}},\tau)}{\partial x^2} = 0 \\
\mathbf{w}(\bar{\mathbf{x}},0,\tau) = \mathbf{w}(\bar{\mathbf{x}},1,\tau) = 0 \quad \left| \frac{\partial^2 \mathbf{w}(\bar{\mathbf{x}},0,\tau)}{\partial y^2} = \frac{\partial^2 \mathbf{w}(\bar{\mathbf{x}},b,\tau)}{\partial y^2} = 0 \right.\n\end{aligned}
$$

Figure 1. Schematic Diagram of a Thin Film at Simple Harmonic Forces\n3. Functional Approximation of The Shift Legendre Polynomial Definition 1: The Legendre polynomial, defined on interval [0,1], is expressed as:\n
$$
l_{n,i}(x) = \sum_{i=0}^{n} (-1) \frac{\Gamma(n+i+1)}{\Gamma(n-i+1)(\Gamma(i+1))^2} x^i \tag{3}
$$
\nWhere $i = 0, 1, ..., n, x \in [0,1]$. The column vector $\varphi(x)$ is constituted by the Legendre polynomial on the interval [0,1], and it can be expressed as\n
$$
\varphi(x) = [l_{n,0}(x), l_{n,1}(x), \ldots, l_{n,n}(x),]^T \tag{4}
$$
\n
$$
= AZ(x)
$$
\nWhere $Z(x) = [1, x, \cdots, x^n]^T$,\nA is the Legendre polynomial coefficient matrix:\n
$$
A_n = \begin{bmatrix} a_{0,0} & 0 & \cdots & 0 \\ a_{1,0} & a_{1,1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \end{bmatrix}, a_{ij}
$$

$$
\varphi(x) = [l_{n,0}(x), l_{n,1}(x), \ldots, l_{n,n}(x),]^T
$$
 (4)
= $AZ(x)$

,

Where $Z(x) = [1, x, \dots, x^n]^T$,

A is the Legendre polynomial coefficient matrix:

$$
= \sum_{i=0} (-1) \frac{I(n+i+1)}{\Gamma(n-i+1)(\Gamma(i+1))^2} x^{i}
$$
\nWhere $i = 0, 1, ..., n, x \in [0, 1]$.
\nThe column vector $\varphi(x)$ is constituted by the
\nLegendre polynomial on the interval [0,1], and
\nit can be expressed as
\n $\varphi(x)$
\n $= [l_{n,0}(x), l_{n,1}(x), ..., l_{n,n}(x),]^T$ (4)
\n $= AZ(x)$
\nWhereZ(x) = [1, x, ..., xⁿ]^T,
\nA is the Legendre polynomial coefficient
\nmatrix:
\n $A_n = \begin{bmatrix} a_{0,0} & 0 & \cdots & 0 \\ a_{1,0} & a_{1,1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,0} & a_{n,1} & \cdots & a_{n,n} \end{bmatrix}, a_{ij}$

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\n $= \begin{cases}\n(-1)^{i+j} \frac{\Gamma(n+i+1)}{\Gamma(n-i+1)\Gamma(i+1)}; & i \geq j \\ 0, i < j.\n\end{cases}$ \n <p>Extending the Legendre polynomial from the integrand $[0,1]$ to $[0,K]$ yields the shifted Legendre polynomial:</p> \n <p>Legendre polynomial:</p> \n <p>Interval $[0,1]$ to $[0,K]$ yields the shifted Legendre polynomial:</p> \n <p>$L_{n,i}(x)$</p> \n <p>$= \sum_{i=0}^{n} (-1)^{n+i} \frac{\Gamma(n+i+1)}{\Gamma(n-i+1)\Gamma(i+1)} \left(\frac{1}{K}\right)^{i} x^{i}$\n<p>Where $i = 0,1,...,n, x \in [0,K]$.</p>\n<p>Where $i = 0,1,...,n, x \in [0,K]$.</p>\n<p>At this point, $\varphi(x)$ can be represented by the displaced Legendre multinomial:</p>\n<p>$\varphi(x) = PZ(x)$</p>\n<p><i>P</i> is the matrix of shifted Legendre polynomial</p>\n<p>coefficients:</p>\n<p>$[p_{0,0} \quad 0 \quad \cdots \quad 0 \quad 1$</p>\n<p>$= \sum_{i=0}^{n} (-1)^{i+j} \frac{\Gamma(n+i+1)}{\Gamma(n-i+1)\Gamma(i+1)} \left(\frac{1}{K}\right)^{i} x^{i}$\n<p>where $i = 0,1,...,n, x \in [0,K]$.</p>\n<p>$= D^{\alpha(t)} \left[\left(\frac{\partial w}{\partial x}\right)^{2} \frac{\partial^{2}w}{\partial x^{2}} + \left(\frac{\partial w}{\partial y}\right)^{2} \frac{\partial^{2}w}{\partial x^{2}} + \left(\frac{\partial w}{$</p></p></p>		

$$
= \begin{cases}\n\begin{cases}\n\cdot & f(n-i+1)(\Gamma(i+1))^{2} \\
0, i < j.\n\end{cases} & \text{as infinite} \\
\text{Extending the Legendre polynomial from the interval [0,1] to [0,K] yields the shifted Legendre polynomial: \\
L_{n,i}(x) - \left(\frac{\partial w}{\partial x}\right)^{n+i} \\
\cdot & \text{where } v = 0,1,\ldots,n, x \in [0,K].\n\end{cases} \\
\text{Where } i = 0,1,\ldots,n, x \in [0,K].
$$
\nAt this point, $\varphi(x)$ can be represented by the displaced Legendre multinomial: $\varphi(x) = PZ(x)$ (6)\n\nP is the matrix of shifted Legendre polynomial coefficients: $[p_{00} \quad 0 \quad \cdots \quad 0 \quad 1]$

$$
\varphi(x) = PZ(x) \tag{6}
$$

coefficients:

Legendre polynomial:
\n
$$
L_{n,i}(x) = \int_{\frac{1}{i=0}}^{n} (-1) \frac{\Gamma(n+i+1)}{\Gamma(n-i+1)(\Gamma(i+1))^{2}} \left(\frac{1}{K}\right)^{i} x^{i}
$$
\n
$$
= \sum_{i=0}^{n} (-1) \frac{\Gamma(n+i+1)}{\Gamma(n-i+1)(\Gamma(i+1))^{2}} \left(\frac{1}{K}\right)^{i} x^{i}
$$
\nWhere $i = 0,1,...,n, x \in [0, K]$.
\nAt this point, $\varphi(x)$ can be represented by the
\ndisplaced Legendre multinomial:
\n $\varphi(x) = PZ(x)$ (6)
\n*P* is the matrix of shifted Legendre polynomial
\ncoefficients:
\n
$$
P = \begin{bmatrix} p_{0,0} & 0 & \cdots & 0 \\ p_{1,0} & p_{1,1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ p_{n,0} & p_{n,1} & \cdots & p_{n,n} \end{bmatrix}, p_{ij}
$$
\nThe boundary cc
\nAmong them [0,1], $y \in [0,1]$.
\n
$$
= \begin{cases} (-1)^{i+j} \frac{\Gamma(n+i+1)}{\Gamma(n-i+1)(\Gamma(i+1))^{2}} \left(\frac{1}{K}\right)^{i}, i \geq j \\ 0, i < j \end{cases}
$$
\nThe governing equation (2) for the thin film
\ncan be expressed as a matrix of variable-order differential operators:
\n
$$
\rho A \varphi^{T}(x) W K_{i}^{2} \varphi(t) + \left(EI + \mu A I^{2}\right) \varphi^{T}(x) M_{x}^{4} W \varphi(t) + \left(EI + \mu A I^{2}\right) \varphi^{T}(x) M_{x}^{4} W \varphi(t)
$$

 $\begin{aligned}\n&\begin{vmatrix}\n\vdots & \vdots & \ddots & \vdots \\
p_{n,0} & p_{n,1} & \cdots & p_{n,n}\n\end{vmatrix}^T \\
&= \begin{cases}\n(-1)^{i+j} \frac{\Gamma(n+i+1)}{\Gamma(n-i+1)(\Gamma(i+1))^2} \left(\frac{1}{K}\right)^i, i \geq 0 \\
0, i < j\n\end{cases} \\
\text{The governing equation (2) for the thin fill can be expressed as a matrix of variable-ord differential operators: \\
&\rho A \varphi^T(x) W K_t^2 \varphi(t) + (EI + \mu A l^2) \varphi^T(x) N_x^4 W \varphi(t) + (\mu A l^2)$... $p_{n,n}$
 $\Gamma(n+i+1)$
 $i-i+1)(\Gamma(i+1))^2\left(\frac{1}{K}\right)^i, i \ge j$
 $0, i < j$

equation (2) for the thin film

d as a matrix of variable-order

ators:
 Γ
 $+ (\mathcal{E}I + \mu A l^2) \varphi^T(x) N_x^4 W \varphi(t)$
 $U^T H^T W^T N_x \varphi(x) \varphi^T(x) N_x W$ [(7)
 $[V^T$ $\begin{aligned}\n&= \begin{cases}\n(-1)^{i+j} \frac{\Gamma(n+i+1)}{\Gamma(n-i+1)\Gamma(i+1)} \left(\frac{1}{K}\right)^{i}, i \geq j \\
0, i < j\n\end{cases} < i\n\end{cases}$ The governing equation (2) for the thin film

can be expressed as a matrix of variable-order

differential operators:
 $\rho A \varphi^T(x) W K_i$ The governing equation (2) for the

can be expressed as a matrix of varia

differential operators:
 $\rho A \varphi^T(x) W K_t^2 \varphi(t)$
 $+ (EI + \mu A l^2) \varphi^T(x) N_x^4 W \varphi(t)$
 $+ [(EI + \mu A l^2) \eta_d] \{Z^T(t) U^T H^T W^T N_x \varphi(x) \varphi^T(x) \}$
 $- \frac{3}{2} E A [\varphi$ $T(t)U^TH^TW^TN_x\varphi(x)\varphi^T(x)N_xW$ [₍₇₎ + $\mu A l^2 \eta_d$ { $Z^T(t) U^T H^T W^T N_x \varphi(x) \varphi^T(x) N_x W$ [. (7)
- $\frac{3}{2} E A [\varphi^T(x) (PSP^{-1})^4 W H U H^{-1} \varphi(t)]$ The governing equation (2) for the thin film

can be expressed as a matrix of variable-order

differential operators:
 $\rho A \varphi^T(x) W K_t^2 \varphi(t)$
 $+ (EI + \mu A l^2) \varphi^T(x) N_x^4 W \varphi(t)$
 $+ [(EI + \mu A l^2) \eta_d] \{Z^T(t) U^T H^T W^T N_x \varphi(x) \varphi^T(x)$ $-\frac{3}{2}E A \eta_d [\varphi^T(t) W^T N_x \varphi(x) \varphi^T(x) N_x W \varphi(t)] [\varphi^T$ $\frac{1}{2}E A \eta_d [\varphi^T(t) W^T N_x \varphi(x) \varphi^T(x) N_x W \varphi(t)]$ perators:
 $\varphi(t)$
 $+ (EI + \mu A l^2) \varphi^T(x) N_x^4 W \varphi(t)$
 $+ (EI + \mu A l^2) \varphi^T(x) N_x^4 W \varphi(t)$
 $- y(2 - 12x + 12x^2) y^2 (1 - 6x^2 + 4x$
 $- y)^2 t^2]^{2} (z$
 $+ 12x^2 y)^2 (t) y^T N_x \varphi(x) \varphi^T(x) N_x W[l]$
 (7)
 $- [x^2 (l - x)^2 (2y - 6y^2 + 4y^2) y^2 (1 + 12x$ $\rho A \varphi^I(x) W K_t^2 \varphi(t)$
 $+ (EI$
 $+ \mu A l^2) \varphi^T(x) N_x^4 W \varphi(t)$
 $+ [(EI$
 $+ \mu A l^2) \eta_d] \{Z^T(t) U^T H^T W^T N_x \varphi(x) \varphi^T(x) N_x W [(7)$
 $- \frac{3}{2} EA [\varphi^T(x) (PSP^{-1})^4 W H U H^{-1} \varphi(t)]$
 $- \frac{3}{2} E A \eta_d [\varphi^T(t) W^T N_x \varphi(x) \varphi^T(x) N_x W \varphi(t)] [\varphi^i$
 $+ c_0 \var$ + $\mu A l^2 \rho \sigma^T(x) N_x^4 W \varphi(t)$

+ $[\mu A l^2 \eta_d] \{Z^T(t) U^T H^T W^T N_x \varphi(x) \varphi^T(x) N_x W$ [(7)

- $\frac{3}{2} E A [\varphi^T(x) (PSP^{-1})^4 W H U H^{-1} \varphi(t)]$

- $\frac{3}{2} E A \eta_d [\varphi^T(t) W^T N_x \varphi(x) \varphi^T(x) N_x W \varphi(t)] [\varphi^1$

+ $c_0 \varphi^T(x) W K_t \varphi(t) = F(x, t)$

Where $W = \begin$ 0,
 $y = -y(2 - 12)$
 $y = -y$ + (EI

+ $\mu A l^2 \rho^T(x) N_x^4 W \varphi(t)$

U^TH^TW^TN_x $\varphi(x) \varphi^T(x) N_x W$ [(7) - $[x^2(1-x)^{p-1}A^4 W H U H^{-1} \varphi(t)]$
 $r^T N_x \varphi(x) \varphi^T(x) N_x W \varphi(t)$][φ^i
 $t) = F(x, t)$
 $W_{0,0}$ $W_{0,1}$... $W_{0,n}$
 $W_{1,0}$ $W_{1,1}$... $W_{1,n}$
 $\$ $\mu A l^2 \rho^d(x) N_x^* W \varphi(t)$
 $\mu^T H^T W^T N_x \varphi(x) \varphi^T(x) N_x W$ [(7) $-[x^2(l-x)^2]$
 $\mu_x \varphi(x) \varphi^T(x) N_x W \varphi(t)] [\varphi^i$
 $\mu = F(x, t)$
 μ_0 $W_{0,1}$ \cdots $W_{0,n}$] \cdots $W_{1,n}$
 \vdots \vdots \ddots \vdots μ_0 $W_{n,1}$ \cdots $W_{n,n}$]
 $\$ $U^T H^T W^T N_x \varphi(x) \varphi^T(x) N_x W$ [(7) $-[x^2(l - x)]^T W H U H^{-1} \varphi(t)]$
 $V^T N_x \varphi(x) \varphi^T(x) N_x W \varphi(t)] [\varphi^T$
 $t) = F(x, t)$
 $W_{0,0} W_{0,1} \cdots W_{0,n}$
 $W_{1,0} W_{1,1} \cdots W_{1,n}$
 $\vdots \vdots \ddots \vdots$
 $W_{n,0} W_{n,1} \cdots W_{n,n}$
 $W_{n,1} W_{n,1} \cdots W_{n,n}$
 \vdots $\begin{array}{llll} &&\hspace{0.5cm} & + \mu A^2 \partial \varphi^T(x) N_x^4 W \varphi(t) & - y)^2 t^2]^2 \\ &&\hspace{0.5cm} (EI &&+ \mu A l^2) \varphi^T(x) N_x W U \\ &&\hspace{0.5cm} & - \big[x^2 (I - x)^2 (2y - 6y^2 + 12x^2) y^2 \\ &&\hspace{0.5cm} \big[2 E A [\varphi^T(x) (PSP^{-1})^4 W H U H^{-1} \varphi(t)] & - \big[x^2 (I - x)^2 (2y - 6y^2 + 12x^2) y^2 \\ &&\hspace{0$ $\frac{3}{2}EA[\varphi^T(x)(PSP^{-1})^4WHUH^{-1}\varphi(t)]$
 $\frac{3}{2}EA\eta_d[\varphi^T(t)W^TN_x\varphi(x)\varphi^T(x)N_xW\varphi(t)][\varphi^T(t)W_K\varphi(t))]$
 $\cdot c_0\varphi^T(x)WK_t\varphi(t) = F(x,t)$
 $\cdot \text{where} \quad W = \begin{bmatrix} W_{0,0} & W_{0,1} & \cdots & W_{0,n} \\ W_{1,0} & W_{1,1} & \cdots & W_{1,n} \\ \vdots & \vdots & \ddots & \vdots \\ W_{n,0} & W_{n,1} & \$ EA $\eta_d[\varphi^T(t)W^TN_x\varphi(x)\varphi^T(x)N_xW\varphi(t)]$
 $\varphi^T(x)WK_t\varphi(t) = F(x,t)$

ere $W = \begin{bmatrix} W_{0,0} & W_{0,1} & \cdots & W_{0,n} \\ W_{1,0} & W_{1,1} & \cdots & W_{1,n} \\ \vdots & \vdots & \ddots & \vdots \\ W_{n,0} & W_{n,1} & \cdots & W_{n,n} \end{bmatrix}$

(i)
 $\eta, i = j, i \neq 1$
 $H = \begin{bmatrix} 0, \text{otherwise} & 0 & \cdots & 0 \\$ $\begin{aligned}\n\begin{aligned}\n\frac{\partial}{\partial x} \rho(x) \varphi^T(x) N_x W \varphi(t) \end{aligned}\n\end{aligned}$ $\begin{aligned}\n\frac{\partial}{\partial x} \rho(x) \varphi^T(x) N_x W \varphi(t) \end{aligned}$ $\begin{aligned}\n\frac{\partial}{\partial x} \rho(x) \varphi^T(x) N_x W \varphi(t) \end{aligned}$ $\begin{aligned}\n\frac{\partial}{\partial y} \rho(x) \cdot \frac{\partial}{\partial y} N_{1,1} &\cdots \quad \frac{\partial}{\partial y} N_{1,1} &\cdots \quad \frac{\partial}{\partial y} N_{1,1} &\cdots$ $h_{0,0}$ 0 \cdots 0 $h_{1,0}$ $h_{1,1}$ \cdots 0 $N_{\rm u}$ is the first-order here $W = \begin{bmatrix} w_{0,0} & w_{0,1} & \cdots & w_{0,n} \\ w_{1,0} & w_{1,1} & \cdots & w_{1,n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n,0} & w_{n,1} & \cdots & w_{n,n} \end{bmatrix}$, $w_{ij} =$
 $\frac{\Gamma(i)}{(i+1)}, i = j, i \neq 1$, $H =$

0, otherwise

0,0 0 … 0

1,0 h_{1,1} … 0
 \vdots \vdots \ddots \vdots \vdots Where $W = \begin{bmatrix} w_{1,0} & w_{1,1} & \cdots & w_{1,n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n,0} & w_{n,1} & \cdots & w_{n,n} \end{bmatrix}$, $w_{ij} = \frac{\sum_{i=1}^{r(i)} j_i}{r(i+1)}, i = j, i \neq 1, H = 0$, otherwise
 $h_{0,0} = 0 \qquad \cdots = 0$
 $h_{1,0} = h_{1,1} \qquad \cdots = 0$
 $h_{n,0} = h_{n,1} \qquad \cdots = h_{n,n}$
 N_x is the first-order $\begin{aligned}\n &\frac{-\frac{1}{2}E A \eta_d(\varphi^*(t)W^* N_x \varphi(t)) - F(x, t) \qquad -12y}{\varphi(\varphi^T(x)W K_t \varphi(t))} + \varphi(\varphi(t)) - F(x, t) \qquad -12y\n \end{aligned}$ Where $W = \begin{bmatrix}\n w_{1,0} & w_{0,1} & \cdots & w_{0,n} \\
 w_{1,1} & w_{1,1} & \cdots & w_{1,n} \\
 \vdots & \vdots & \ddots & \vdots \\
 w_{n,0} & w_{n,1} & \$ + $c_0 \varphi^T(x) W K_t \varphi(t) = F(x, t)$

Where $W = \begin{bmatrix} w_{0,0} & w_{0,1} & \cdots & w_{0,n} \\ w_{1,0} & w_{1,1} & \cdots & w_{1,n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n,0} & w_{n,1} & \cdots & w_{n,n} \end{bmatrix}$, $w_{ij} = -[(2x - 6x^2 + 4x^3 - 4x^3 + 1 + 1 - 4x^2 - 4x^2 + 4x^3 + 4x^2 + 4x^3 + 4x^2 + 4x^3 + 4x$ Where $W = \begin{bmatrix} w_{0,0} & w_{0,1} & \cdots & w_{0,n} \\ w_{1,0} & w_{1,1} & \cdots & w_{1,n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n,0} & w_{n,1} & \cdots & w_{n,n} \end{bmatrix}$, $w_{ij} = \begin{bmatrix} -(2x - 6x^2 + 4x^3)y^2 & -x^2(y^2 + 4x^3)y^3 & -x^2(y^2 + 4x^3)y^2 & -x^2(y^2 + 4x^3)y^2 & -x^2(y^2 + 4x^3)y^3 & -x^2(y^2 + 4x$ Where $W = \begin{bmatrix} w_{1,0} & w_{1,1} & \cdots & w_{1,n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n,0} & w_{n,1} & \cdots & w_{n,n} \end{bmatrix}$, $w_{ij} = -[(2x - 6x^2 + 4x^3 + 4x^3 + 4x^2 + 4x^3 + 4x^3 + 4x^2 + 4x^3 + 4x$ $\begin{cases}\n\frac{f'(t)}{f'(t+1)}, i = j, i \neq 1, H = 0 \\
0, otherwise\n\end{cases}$
 $\begin{cases}\nh_{0,0} & 0 & \cdots & 0 \\
h_{1,0} & h_{1,1} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
h_{n,0} & h_{n,1} & \cdots & h_{n,n}\n\end{cases}$, N_x is the first-order $\begin{cases}\n-k & \to i+4y^2 + 4y^3 + 12y^2 + 12y^2 \\
\vdots & \vdots \\
k & \text{otherwise}\n\end$

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Science and Engineering Vol. 1 No. 10, 2024
that the parameters in numerical studies are
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 $\$

Accatemic Education	Mdustry Science and Engineering Vol. 1 No. 10, 2024
\n $= \begin{cases}\n(-1)^{i+j} \frac{\Gamma(n+i+1)}{\Gamma(n-i+1)\Gamma(i+1)^2}, i \geq j & \text{that the parameters in numerical studies are arbitrary values and have no actual physical significance. Numerical calculations are the same as above.\n $	
\n Extending the Legendre polynomial from the interval [0,1] to [0,K] yields the shifted\n	\n $\frac{\partial^2 w}{\partial t^2} + \frac{\partial^2 w}{\partial x \partial t^2} + \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2$

 $F(x, y, t) = 2x^2(1 - x)^2y^2(1 - y)^2$ K/T , ≥ We the thin film
 $P = \begin{cases} p_{0,0} & 0 & \dots & 0 \\ 0 & p_{1,1} & \dots & p_{n,2} \\ 0 & p_{1,2} & \dots & p_{1,n} \end{cases}$
 $P = \begin{cases} p_{0,0} & 0 & \dots & 0 \\ 0 & p_{1,1} & \dots & p_{1,n} \\ 0 & p_{n,1} & \dots & p_{n,n} \end{cases}$
 $P = \begin{cases} p_{0,0} & 0 & \dots & 0 \\ 0 & p_{1,1} & \dots & p_{1,n} \\ 0 & p_{n,1} & \dots & p_{n,n} \end{$ (
 $\begin{aligned}\n & (0, i < j) \\
 & (0, i < j) \\
 & (0, i < j) \\
 & (0, i < j)\n \end{aligned}$

The governing equation (2) for the thin film
 $\begin{aligned}\n & (0, i < j) \\
 & (0, i < j) \\
 & (0, i < j) \\
 & (0, i < j)\n \end{aligned}$
 $\begin{aligned}\n & (0, i < j) \\
 & (0, i < j) \\
 & (0, i < j)\n \end{aligned}$
 $\$ (7) $-[x^2]$ $D^{a(t)}\left[\left(\frac{\partial w}{\partial x}\right)^2 \frac{\partial^2 w}{\partial x^2} + \left(\frac{\partial w}{\partial y}\right)^2 \frac{\partial^2 w}{\partial x^2}\right]$
 $+ \left(\frac{\partial w}{\partial y}\right)^2 \frac{\partial^2 w}{\partial y^2}$
 $+ \left(\frac{\partial w}{\partial x}\right)^2 \frac{\partial^2 w}{\partial y^2}\right] = F$

The boundary conditions are the same as above.

Among them are: $\alpha(t) = 1 - 0.7$ + $\left(\frac{\partial w}{\partial y}\right)^2 \frac{\partial^2 w}{\partial y^2}$
+ $\left(\frac{\partial w}{\partial x}\right)^2 \frac{\partial^2 w}{\partial y^2}$ = F
mditions are the same as above.
are: $\alpha(t) = 1 - 0.75t$, $x \in$
 $(1 - x)^2 y^2 (1 - y)^2$
+ $2t (2x - 6x^2 + 4x^3) y^2 (1 - y)^2 + (2x - 6x^2 + 4x^3) y^2 (1 - y)^2 t + 0.35$
* $(2$ $2(1$ + $\left(\frac{\partial w}{\partial y}\right)^2 \frac{\partial^2 w}{\partial y^2}$
+ $\left(\frac{\partial w}{\partial \overline{x}}\right)^2 \frac{\partial^2 w}{\partial y^2}$ = F
mditions are the same as above.
are: $\alpha(t) = 1 - 0.75t, x \in$
 $(1 - x)^2 y^2 (1 - y)^2$
+ $2t (2x - 6x^2 + 4x^3) y^2 (1 - y)^2 + (2x - 6x^2 + 4x^3) y^2 (1 - y)^2 t^2 + 0.35$
* 2 + $\left(\frac{\partial w}{\partial \bar{x}}\right)^2 \frac{\partial^2 w}{\partial \bar{y}^2}$ = F

mditions are the same as above.

are: $\alpha(t) = 1 - 0.75t$, $x \in$
 $(1 - x)^2 y^2 (1 - y)^2$

+ $2t(2x - 6x^2 + 4x^3)y^2 (1 - y)^2 + (2x - 6x^2 + 4x^3)y^2 (1 - y)^2 t^2$

+ $4x^3 y^2 (1 - y)^2 2t + 0.35$

* $(2$ $^{3})y^{2}(1-y)^{2}2t+0.35$ + $\left(\frac{\partial}{\partial x}\right) \frac{\partial}{\partial y^2}$ = F

mditions are the same as above.

are: $\alpha(t) = 1 - 0.75t$, $x \in$
 $(1 - x)^2 y^2 (1 - y)^2$

+ $2t(2x - 6x^2 + 4x^3)y^2 (1 - y)^2 + (2x - 6x^2 + 4x^3)y^2 (1 - y)^2 t^2$

+ $4x^3 y^2 (1 - y)^2 2t + 0.35$

* $(2 - 12x + 12x^2$ $*(2-12x+12x^2)y^2(1$
-y)²t² $2(1$ mditions are the same as above.

are: $\alpha(t) = 1 - 0.75t$, $x \in$
 $(1 - x)^2 y^2 (1 - y)^2$
 $+ 2t(2x - 6x^2 + 4x^3)y^2 (1 - y)^2 + (2x - 6x^2 + 4x^3)y^2 (1 - y)^2 t^2$
 $+ 4x^3 y^2 (1 - y)^2 t^2 + 0.35$
 $*(2 - 12x + 12x^2)y^2 (1 - y)^2 t^2$
 $12x^2)y^2 (1 - y)^2 t^2 - [(2$ t^2 The countary conditions are the same as above.

Among them are: $\alpha(t) = 1 - 0.75t$, $\alpha \in [0,1]$, $y \in [0,1]$.
 $F(x, y, t) = 2x^2(1 - x)^2y^2(1 - y)^2$
 $+ 2t(2x - 6x^2 + 4x^3)y^2(1 - y)^2t + (2x - 6x^2 + 4x^3)y^2(1 - y)^2t^2 + 0.35$
 $+ (2 - 12x + 1$ $(2y^2)(1-y)^2t^2-[(2x)]$ $\alpha(t) = 1 - 0.75t, x \in$
 $\alpha(t) = 1 - 0.75t, x \in$
 $\alpha(t) = 1 - 0.75t, x \in$
 α^2
 $\$ $(1-x)^2y^2(1-y)^2$
 $+ 2t(2x - 6x^2 + 4x^3)y^2(1 - y)^2 + (2x - 6x^2 + 4x^3)y^2(1 - y)^22t + 0.35$
 $*(2-12x + 12x^2)y^2(1 - y)^2t^2$
 $(-y)^2t^2$
 $12x^2)y^2(1 - y)^2t^2 - [(2x - 6x^2 + 4x^3)y^2(1 - y)^2t^2 - 2x + 12x^2)y^2(1 - y)^2t^2$
 $y - 6y^2 + 4y^3)t^2]^2(2 - 12x$ $(1-x)^2y^2(1-y)^2$
+ 2t(2x - 6x² + 4x³)y²(1
- y)² + (2x - 6x²
+ 4x³)y²(1 - y)²2t + 0.35
* (2 - 12x + 12x²)y²(1
- y)²t²
12x²)y²(1 - y)²t² - [(2x
- 6x² + 4x³)y²(1
- y)²t²]²(2 - 12x
+ $\lbrack t^2 \rbrack^2 (2-12x)$ $2y^2(1-y)^2$
 $x-6x^2+4x^3$)y²(1
 $-(2x-6x^2$
 $y^2(1-y)^22t+0.35$
 $(2x+12x^2)y^2(1)$
 $(1-y)^2t^2-[2x^2+4x^3)y^2(1)$
 $]$ ²(2 - 12x
 $)y^2(1-y)^2t^2$
 $+4y^3)t^2$]²(2 - 12x
 $)y^2(1-y)^2t^2-[x^2(1-x)^2-6y^2]$
 $[y-6y^2+2]^2x^2(1-x)^2(2)$ + 2t(2x - 6x² + 4x³)y²(1

- y)² + (2x - 6x²

+ 4x³)y²(1 - y)²2t + 0.35

* (2 - 12x + 12x²)y²(1

- y)²t²

12x²)y²(1 - y)²t² - [(2x

- 6x² + 4x³)y²(1

- y)²t²]²(2 - 12x

+ 12x²) t^2 $- y)^2 + (2x - 6x^2$
 $- y)^2 + (2x - 6x^2$
 $+ 4x^3)y^2(1 - y)^22t + 0.35$
 $*(2 - 12x + 12x^2)y^2(1$
 $- y)^2t^2$
 $- y(2 - 12x + 12x^2)y^2(1 - y)^2t^2 - [(2x - 6x^2 + 4x^3)y^2(1 - y)^2t^2 - [x^2(1 - x)^2(2y - 6y^2 + 4y^3)t^2]^2(2 - 12x$
 $+ 12x^2)y^2(1 - y)^2t^2 - [x^2$ + 4x³)y²(1 - y)²zt + 0.35

*(2 - 12x + 12x²)y²(1

- y)²t²

12x²)y²(1 - y)²t² - [(2x

- 6x² + 4x³)y²(1

- y)²t²]²(2 - 12x

+ 12x²)y²(1 - y)²t² - [x²(l

y - 6y² + 4y³)t²]² $(2^2)y^2(1-y)^2t^2 - [x^2(l)]$ 2x - 3x

(1 − y)²2t + 0.35

x + 12x²)y²(1

l − y)²t² - [(2x

kx³)y²(1

(2 - 12x

²(1 - y)²t²

- 4y³)t²]²(2 - 12x

²(1 - y)²t² - [x²(l

- 6y²

²x²(1 - x)²(2

12y²)t²

- y)²t $l^{2}(l)$ + 1x fy (1 y) 2t + 0.55

* (2 - 12x + 12x²)y²(1

- y)²t²

12x²)y²(1 - y)²t² - [(2x

- 6x² + 4x³)y²(1

- y)²t²]²(2 - 12x

+ 12x²)y²(1 - y)²t² - [x²(l

y - 6y² + 4y³)t²]²(2 - 12 2 $\begin{aligned} &\frac{d}{dx} \frac{d}{dx} \left(\frac{d}{dx} + \frac{d}{dx} \right) y(x) \\ &- y)^2 t^2 \\ &\frac{d}{dx} \frac{d}{dy} \left(\frac{d}{dx} \right) \left(\frac{d}{dx} \right) \\ &- y)^2 t^2 \left(\frac{d}{dx} \right) \left(\frac{d}{dx} \right) \\ &+ 12x^2 \left(\frac{d}{dx} \right) \left(\frac{d}{dx} \right) \left(\frac{d}{dx} \right) \\ &- 6y^2 + 4y^3 \right) t^2 \left(\frac{d}{dx} \right) \left(\frac{d}{dx$ $(x^3)t^2\frac{1}{2}x^2(1-x)^2(2$ $(y)^2t^2 - [(2x)^3)y^2(1 - 12x$
 $(1-y)^2t^2$
 $(y^3)t^2]^2(2 - 12x$
 $(1-y)^2t^2 - [x^2(t^2)]$
 $(y^2)^2(t^2 - 12x^2)$
 $(y^2)t^2$
 $(y^2)t^2$
 $(y^2)t^2$
 $(1-x)^2(t^2)$
 $(y^2)t^2$
 $12x^2)y^2(1-y)^2t^2 - [(2x$
 $- 6x^2 + 4x^3)y^2(1$
 $-y)^2t^2]^2(2 - 12x$
 $+ 12x^2)y^2(1 - y)^2t^2$
 $y - 6y^2 + 4y^3)t^2]^2(2 - 12x$
 $+ 12x^2)y^2(1 - y)^2t^2 - [x^2(l - x)^2(2y - 6y^2 + 4y^3)t^2]^2x^2(1 - x)^2(2$
 $- 12y + 12y^2)t^2$
 $4x^3)y^2(1 - y)^2t^2]^2x^2$ $y(z)$
 $-6x^2 + 4x^3y^2(1$
 $-y)^2t^2]^2(2 - 12x$
 $+ 12x^2y^2(1 - y)^2t^2$
 $-[x^2(l - x)^2(2y - 6y^2 + 4y^3)t^2]^2(2 - 12x$
 $+ 12x^2y^2(1 - y)^2t^2 - [x^2(l - x)^2(2y - 6y^2 + 4y^3)t^2]^2x^2(1 - x)^2(2$
 $- 12y + 12y^2)t^2$
 $-[[(2x - 6x^2 + 4x^3)y^2(1 - y)^2t^2$ $\left[1\right]^{2}x^{2}\left(1\right)$ $(x + 12x^2)y^2(1 - y)^2t^2$
 $y - 6y^2 + 4y^3)t^2$ $y^2(1 - y)^2t^2$
 $y - 6y^2 + 4y^3)t^2$ $y^2(1 - y)^2t^2 - [x^2(l - x)^2(2y - 6y^2 + 4y^3)t^2]^{2}x^2(1 - x)^2(2y - 12y + 12y^2)t^2$
 $y^2(1 - y)^2t^2$ $y^2(1 - y)^2t^2$ $y^2(1 - x)^2(2 - 12y + 12y^2)t^2$
 $y^2(1 - y)^2$ $- [x^2(l-x)^2(2y-6y^2+4y^3)t^2]^2(2-12x$
 $+ 12x^2y^2(1-y)^2t^2 - [x^2(l$
 $+ 2x^2y^2(1-y)^2t^2 - [x^2(l$
 $-x)^2(2y-6y^2$
 $+ 4y^3)t^2]^2x^2(1-x)^2(2$
 $- 12y + 12y^2)t^2$
 $-[[(2x-6x^2+4x^3)y^2(1-y)^2t^2]^2x^2(1 - x)^2(2-12y+12y^2)t^2$
 $-[[(2x-6x^2+4x^$ $y = 6y^2 + 4y^3(t^2)t^2$
 $y = 6y^2 + 4y^3(t^2)t^2$
 $y = 12x^2y^2(1 - y)^2t^2 - [x^2(t^2 - x^2)(2y - 6y^2 + 4y^3)t^2]^2x^2(1 - x)^2(2x^2 - 12y + 12y^2)t^2$
 $4x^3y^2(1 - y)^2t^2]^2x^2(1 - x)^2(2 - 12y + 12y^2)t^2$
 $4x^3y^2(1 - y)^2]^2(2 - 12x + 12x^2)y^2(1 - y)^2\frac{\$ $^{2})y^{2}(1$ $y = 0$
 $y = 4y^2$ $y^2(1 - y)^2t^2 - [x^2(1 - x)^2(2y - 6y^2 + 4y^3)t^2]^2x^2(1 - x)^2(2 - 12y + 12y^2)t^2$
 $= 12y + 12y^2)t^2$
 $= x^3y^2(1 - y)^2t^2y^2t^2(1 - x)^2(2 - 12y + 12y^2)t^2$
 $= x^3y^2(1 - y)^2y^2(2 - 12x + 12x^2)y^2(1 - y)^2y^2(1 - y)^2\frac{\Gamma(7)}{\Gamma(7 - at$ $(-y)^2 \frac{\Gamma(7)}{\Gamma(7-at)} t^{(6-at)}$ + 4y³)t²]²x²(1 - x)²(2

- 12y + 12y²)t²

4x³)y²(1 - y)²t²]²x²(1

- x)²(2 - 12y + 12y²)t²

4x³)y²(1 - y)²]²(2 - 12x

+ 12x²)y²(1

- y)² $\frac{\Gamma(7)}{\Gamma(7 - at)}t^{(6-at)}$

- [x²(1 - x) 2 + 1y y, c + 2

+ 2y + 12y²)t²

+ 2y + 12y²)t²

+ 2y²(2 − 12y + 12y²)t²

+ 2x²)y²(1 − y)²]²(2 − 12x

+ 12x²)y²(1

− y)² $\frac{\Gamma(7)}{\Gamma(7 - at)}$ t^(6-at)

− [x²(1 − x)²(2y − 6y²

+ 4y³)]²(2 − $4x^3y^2(1-y)^2t^2\frac{1}{2}x^2(1-x)^2(2-12y+12y^2)t^2$
 $4x^3y^2(1-y)^2\frac{1}{2}(2-12x+12x^2)y^2(1-y)^2\frac{\Gamma(7)}{\Gamma(7-at)}t^{(6-at)}$
 $- [x^2(1-x)^2(2y-6y^2+4y^3)]^2(2-12x+12x^2)y^2(1-y)^2\frac{\Gamma(7)}{\Gamma(7-at)}t^{(6-at)}$
 $y-6y^2+4y^3)]^2x^2(1-y)^2\frac{\Gamma(7)}{\Gamma(7-at)}t^{(6-at)}$ $^{2})y^{2}(1$ $(x + 2)$ $y^2 (1 - y)$ $y^2 (1 - 2)$
 $(x^2 - 2)^2 (2 - 12y + 12y^2) t^2$
 $(x^3)y^2 (1 - y)^2$ $y^2 (1 - y)^2$
 $(x^2 - 2)^2 (1 - x)^2 (2y - 6y^2)$
 $(x^2 - 2)^2 (2y - 6y^2)$
 $(x^2 + 4y^3)$ $y^2 (2 - 12x)$
 $(x^2 + 12x^2) y^2 (1 - y)^2$
 $\frac{\Gamma(7)}{\Gamma(7 - at)} t^{(6 - at)}$
 $y \frac{2}{(1-y)^2}$
 $\frac{2}{(1-y)^2}$
 $\frac{1}{(1-x)^2}$
 $\frac{1}{(1-x)^2}$
 $\frac{(1-x)^2(2y-6y^2)}{(2-12x)}$
 $\frac{1}{(1-x)^2}$
 $(-y)^2 \frac{\Gamma(7)}{\Gamma(7-at)} t^{(6-at)}$ $+12x^2y^2$ + $\frac{\Gamma(7)}{\Gamma(7 - at)} t^{(6-at)}$
 $- [x^2(1-x)^2(2y-6y^2 + 4y^3)]^2(2-12x$
 $+ 12x^2)y^2(1 - y)^2 \frac{\Gamma(7)}{\Gamma(7-at)} t^{(6-at)}$
 $-[x^2(1-x)^2(2y-6y^2+4y^3)]^2x^2(1-x)^2(2-12y+12y^2) \frac{\Gamma(7)}{\Gamma(7-at)} t^{(6-at)} - [(2x-6x^2+4x^3)y^2(1-y)^2]^2x^2(1-x)^2(2-12y+1$ $(-6-4)$
 $(-6y^2)$
 $(-6-4)$
 $(-2)(1-$
 $(-2x-$
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 $(-2-2x))$
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 $-y)^2 \frac{\Gamma(7)}{\Gamma(7 - at)} t^{(6-at)}$
 $- [x^2(1 - x)^2(2y - 6y^2 + 4y^3)]^2(2 - 12x + 12x^2)y^2(1 - y)^2 \frac{\Gamma(7)}{\Gamma(7 - at)} t^{(6-at)}$
 $-[x^2(1 - x)^2(2y - 6y^2 + 4y^3)]^2x^2(1 - x)^2(2 - 12y + 12y^2) \frac{\Gamma(7)}{\Gamma(7-at)} t^{(6-at)} - [(2x - 6x^2 + 4x^3)y^2(1 - y)^2]^2x^2(1 - x)^2(2 - 12y +$ 7− (6−at)

(2) $t^{(6-at)}$
 $(2y - 6y^2)$
 $(12x)$
 $(t^{(6-at)} - t^{(6-at)} - [(2x -$
 $(x - x)^2(2 -$

e exact solution $\Gamma(7 - at)^3$
 $- [x^2(1 - x)^2(2y - 6y^2 + 4y^3)]^2(2 - 12x$
 $+ 12x^2)y^2(1$
 $- y)^2 \frac{\Gamma(7)}{\Gamma(7 - at)} t^{(6-at)}$
 $-[x^2(1 - x)^2(2y - 6y^2 + 4y^3)]^2x^2(1 - x)^2(2 - 12y + 12y^2) \frac{\Gamma(7-at)}{\Gamma(7-at)} t^{(6-at)} - [(2x - 6x^2 + 4x^3)y^2(1 - y)^2]^2x^2(1 - x)^2(2 - 12y + 12y^2)$ $\int_0^2 x^2 (1-x)^2 (2-x)^2$ $(7 - at)^2$
 $- x)^2 (2y - 6y^2$
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 $(6 - at)^2$
 $(6 - at)^2$
 $(1 - x)^2 (2 - at)^2$
 $(1 - x)^2 (2 - at)^2$

(b) The exact solution $- [x^2(1-x)^2(2y - 6y^2 + 4y^3)]^2(2 - 12x$
 $+ 12x^2)y^2(1$
 $- y)^2 \frac{\Gamma(7)}{\Gamma(7 - at)} t^{(6-at)}$
 $-[x^2(1-x)^2(2y - 6y^2 + 4y^3)]^2x^2(1-x)^2(2-12y + 12y^2) \frac{\Gamma(7)}{\Gamma(7-at)} t^{(6-at)} - [(6x^2 + 4x^3)y^2(1-y)^2]^2x^2(1-x)^2(2-12y + 12y^2) \frac{\Gamma(7)}{\Gamma(7-at)} t^{(6-at)}$ The $-y)^2 \frac{I(7)}{\Gamma(7 - at)} t^{(6-at)}$
 $(2y - 6y^2 + 4y^3)]^2 x^2 (1 - 4y^2) + 12y^2 \frac{I(7)}{I(7-at)} t^{(6-at)} - [6(1-y)^2]^2 x^2 (1-x)^2 (2 - 4y^2) + 12y^2 \frac{I(7)}{I(7-at)} t^{(6-at)}$ The exact so x^2) y^2 (1
 $\frac{1}{\Gamma(7 - at)} t^{(6-at)}$
 x^2 (1 - x)²(2y - 6y²
 x^3)]²(2 - 12x
 x^2) y^2 (1
 $\frac{1}{\Gamma(7-at)} t^{(6-at)}$
 $\frac{1}{\Gamma(7-at)} t^{(6-at)} - [(2x -$
 $x^2]^2 x^2 (1 - x)^2 (2 -$
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 $x^2 (1 - x)^2 (2 -$
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Industry Science and Engineering Vol. 1 No. 10, 2024

Industry Science and Engineering Vol. 1 No. 10, 202
of the equation is:
 $w(x, y, t) = x^2(1 - x)^2y^2(1 - y)^2t^2$ (9)
When $n = 2$, the shift Legendre polynomial
algorithm is used to solve the equation (2). the
numerical solution ustry Science and Engineering Vol. 1 No. 10, 2

he equation is:
 $w(x, y, t) = x^2(1 - x)^2y^2(1 - y)^2t^2$ (9)

en $n = 2$, the shift Legendre polynomial

orithm is used to solve the equation (2). the

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Fig. 2

²(1 - y)²t² (9) solutio

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the equation (2). the As can

performed using numerical solution, Legendre mumerical solution, Legendre polynomial solutio $w(x, y, t) = x^2(1-x)^2y^2(1-y)^2t^2$ (9) **Industry Science and Engineering Vol. 1 No. 10, 2024**

of the equation is:
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When $n = 2$, the shift Legendre polynomial respectively.

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of the equation is:
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of the equation is:
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of the equation is:
 $w(x, y, t) = x^2(1-x)^2y^2(1-y)^2t^2$ (9) solution and at

When $n = 2$, the shift Legendre polynomial respectively.

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of the equation is:
 $w(x, y, t) = x^2(1-x)^2y^2(1-y)^2t^2$ (9) solution and absolute

When $n = 2$, the shift Legendre polynomial respectively.

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of the equation is:
 $w(x, y, t) = x^2(1-x)^2y^2(1-y)^2t^2$ (9)
When $n = 2$, the shift Legendre polynomial
algorithm is used to solve the equation (2). the
numerical solution was perf

$$
e(x, t) = |w(x, t) - w_n(x, t)| \tag{10}
$$

respectively.

(a)Numerical solution

lays the theoretical foundation content of a Numerical Study

5. Conclusion

In this paper, the variable fractional nonlinear

differential equations for viscoelastic films are

established, and a nu (a)Numerical solution

(a) and the solution

(a) and the solution

Figure 2. Solution of a Numerical Study

5. Conclusion

In this paper, the variable fractional nonlinear

solutions for viscoelastic films are

solutions (a)Numerical solution

Figure 2. So

5. Conclusion

In this paper, the variable fractional nonlinedifferential equations for viscoelastic films

established, and a numerical algorithm

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In this paper, the variable fractional nonlinear

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