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A new LF for Switched Systems with Sampled Control

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Abstract: This study introduces a novel looped-function (LF) approach for evaluating the stability of linear switching systems under sampled-triggered control. The method introduced a novel stability of system and based on the discrete-time Lyapunov theorem, to present a sampling-triggered for switching systems. It greatly reduces the conservation of the system and the method is based on the discrete-time Lyapunov theorem. Based on the LF, it can improve stable criteria for switched systems with sampled control. The combination of LF and switching system can greatly reduce the amount of calculation. Therefore, our theoretical analysis is great significance and greatly promote the stability of the control system.

KeyWords: LF; Sampled Control; Switching System

1. Introduction

Over the past several years, switching systems have garnered increasing attention and popularity in various fields of research and application [1]. In our daily life, aerospace, military science and technology have a large number of applications. There are lots of researchers focusing on the re- search of switching systems . In the traditional method, we used Lyapunov method to work out the stability [2]. Lyapunov stability is a general method for stability analysis, multivariate, linear, nonlinear, time-invariant and time-varying systems based on state space description of the system. It not only describes the external characteristics of the system, but also fully reveals the internal characteristics of the system. But the Lyapunov function must be positive and the first derivative must be negative. It is essentially a conservative approach to problem-solving [3].Recently the sampled-data system has attracted growing interest from numerous researchers, primarily due to its extensive applications across various domains, such as Electronic computers, communications, radar, television, automatic control, telemetry and remote control, radio navigation and measurement technology [4]. As is well-known, the traditional Lyapunov function is very important for stability analysis. For the stability the Lyapunov function must be positive and the first derivative must be negative [5].

The LF is a novel stability analysis for system. This paper introduce a novel method analysis for sampled switching system. The LF can be used widely [6-10], for the marker time system it use be important, for example the impulsive system , sampled- date system and it can be used in sampled-data control of Markov jump systems. In this paper the discrete Lyapunov function must be positive but the first derivative don't have to negative. In this paper we define $W(t) = V(t) + V(t,\cdot)$ and $t \in [t_k, t_{k+1})$ [11], and the V (t) is discrete Lyapnov function, V(t) is called LF. W(t) must be satisfied W (t_{k+1}) < W (T_k), and the looped-function V(t, ·) must be satisfied the equation: V > 0 and V(0, ·) = V(T, ·), it's a LF of period satisfied T [12]. To introduce the LF, the monotonicity of the function is relaxed. The application has been greatly improved of the Lyapunov function. Switching, sampled, LF, and when they are combined they made a novel method for the analysis of stability. In traditional methods for stability analysis of switching system, to deal with those system should be strict Lyapunov function. To introduce the LF to deal with the sampled-date switching system can reduce constraint [13].

Inspired from the above problem, we introduce the LF or sampled-date switching system. The LF must be satisfied that the sequence $V(\tau,(\cdot)) > 0$ and $V\left(0,(\cdot) = V(T_k,(\cdot))\right)$, and the function V is non-monotonically. The main advantages for this paper as following:

(1) In section 2, first solving for the liner system $\dot{x}(t)$ we can convert the system to

$\dot{\chi}_k(\tau)$, and $\dot{\chi}_k(\tau)$ can visually get the state relationship in each interval [0, T_k], where k ∈ ℕ for corresponding each subsystem. Then some useful definition for stability analysis are presented, and the import definition LF is defined;
(2) In section 3, the impulsive mechanism and controller is by given. By LF proved stability with the mechanism and controller system;
(3) The rest of the paper provide the stability of the system by loop-function, and give the conclusions for this paper.
In order to simply analyze the stability of the system, in this paper the switching system is assumed into two cases:
(a) When the sampled occurs, but the switching condition is not met, so it does not switching. For this reason, we assume stability of the subsystem sequence;
(b) When the sampled occurs and the switching conditions are met, the system should also be switched at this time to obtain a new subsystem of the system, in this time the $e_k(\tau)$ error function is zero.
Anyway, if you switch subsystems, you're going to go to the next subsystem, and what happens is that the sampled is going to go to the next sampled, and if don't switch the sampling and subsystem synchronization are not switched into next sequence.
Notations: In this article, ℕ is set of nonnegative integer numbers, $\mathbb{R}^+$ is set of the nonnegative scalars, ℝ is set of real number, $\mathbb{R}^n$ denote then-dimensional Euclidean space, $\mathbb{R}^{n\times n}$ is n-dimensional matrices space. The notation $|\cdot|$ is defined as an Euclidean norm. λ, λ_{min}, λ_{max} are the corresponding eigenvalue, minimum eigenvalue and maximum eigenvalue, respectively. The superscripts "*T*" and "-1" are represented as transpose and inverse for a matrix. A matrix *P*, *P* > 0 means the matrix is positive and symmetric.

2. Preliminaries

Consider the sampling switched linear systems:

$$\begin{cases} \dot{x}(t) = A_\sigma x(t) + B_\sigma u(t_k), \\ y(t) = C_\sigma x(t), \\ x(0) = x_0. \end{cases} \qquad (1)$$

where x(t) ∈ $\mathbb{R}^m$ is the system state, $x(0) = x_0$ is the initial value, u ∈ $\mathbb{R}^n$ is the control input, y(t) ∈ $\mathbb{R}^p$ is the measurable output, $\{t_k\}_{k\in\mathbb{N}}$ satisfied $t_0 < t_1 < t_2 \cdot\cdot\cdot < t_n$ in sampling instants. σ is the switching signal and it just represents the number of subsystems, when the subsystem is switching,the serial number of σ is increased by one immediately, while it is a piecewise constant function of right continuous and $\sigma : [0, \infty) \to M = \{1, 2, 3 \cdot\cdot\cdot, l\}$, for each switching subsystem A_i , B_i , C_i and i ∈ M are constant matrices. In this paper, the control input is linear then u(t) = $Kx(t_k)$. Then the system (1) is constructed as

$$\begin{cases} \dot{x}(t) = A_\sigma x(t) + B_\sigma K_\sigma x(t_k), \\ y(t) = C_\sigma x(t). \end{cases} \qquad (2)$$

For each sampling interval there is switching with exists at least one sampling in $[t_k, t_{k+1})$k∈ N . There exist two scalars 0 < η 1 < η2 such that

$$T_k = t_{k+1} - t_k \in [\eta_1, \eta_2]. \qquad (3)$$

From (2), we can get

$$\begin{aligned} x(t) &= e^{A_\sigma(t-t_k)}x(t_k) + \int_{t_k}^{t} e^{A_\sigma(t-s)}B_\sigma K_\sigma x(t_k)ds \\ &= e^{A_\sigma(t-t_k)}x(t_k) + \int_0^{t-t_k} e^{A_\sigma(t-t_k-\theta)}B_\sigma K_\sigma x(t_k)d\theta. \end{aligned}$$

Let $\tau = t - t_k$ and $\tau \in [0, T_k]$ then

$$x(t_k+\tau) = e^{A_\sigma|\tau|}x(t-\tau) + \int_0^{\tau} e^{A_\sigma(\tau-\theta)}B_\sigma K_\sigma x(t-\tau)d\theta,$$

which can define as

$$\chi_k(\tau) = e^{A_\sigma\tau}\chi_k(0) + \int_0^{\tau} e^{A_\sigma(\tau-\theta)}B_\sigma K_\sigma \chi_k(0)d\theta,$$

where the K refers to *Kth* subsystem, and τ ∈ $[t_k, t_{k+1}]$. Then the system (2) can construct as

$$\begin{cases} \dot{x}(t) = \dot{\chi}_k(\tau) = A_\sigma \chi_k(\tau) + B_\sigma K_\sigma \chi_k(0), \\ y(t) = C_\sigma x(t) = C_\sigma \chi_k(\tau). \end{cases} \qquad (4)$$

Definition 2.1. define

$$|x(t) - \tilde{x}(t)| < \delta. \qquad (5)$$

This inequality holds, when the system does not satisfy the switching condition, in other words it means that these are approximately equal, where $t \in [t_k, t_{k+1})$, $\tilde{x}(t)$ denote the system state, x(t) denote the sampled system state and δ > 0 is a enough smaller number, respectively.
Definition 2.2. define a function as

$$f: [0, T_{max}] \times K_{[T_{min},T_{max}]} \times [T_{min}, T_{max}] \to \mathbb{R}, \qquad (6)$$

where K is defined as a union set of the continuous function, $\varphi \le T_{min} \le T_{max} < \infty, \varphi > 0$, it is called to be as a LF if those are satisfied
(i)$f(0,\cdot) = f(T,\cdot)$. all T ∈ $[T_{min}, T_{max}]$;
*(ii)*the first variable is differentiable for the function f.
Definition 2.3. If there exists a function V:$\mathbb{R}^n$

$\rightarrow \mathbb{R}^{+}$ such that

(i)$\omega_1|x| \leq V(t,x) \leq \omega_2|x|$;

(ii)$\dot{V}(t,x) <-\ \omega_3|x|$.

Where $\omega_i|_{i=1,2,3}$ are some positive constants. Therefore the solution of (1) is stable.

Definition 2.4. define a function W(τ, χ_k) for all (k, T_k , τ) ∈ ℕ and

$$W(\tau,\chi_k) = V\big(\chi_k(\tau)\big) + V(\tau,\chi_k),$$

then

$$\dot{W}(\tau,\chi_k) = \frac{d}{d\tau}\big[V\big(\chi_k(\tau)\big) + V(\tau,\chi_k)\big], \quad (7)$$

where $V\big(\chi_k(\tau)\big)$ is Lyapunov function and satisfied $V\big(\chi_k(0)\big) > V\big(\chi_k(T_k)\big)$, V is LF and satisfied definition 2.2 .

Theorem 2.1. For $\delta \leq T_{min} \leq T_{max} < \infty$, there exist two positive scalars 0 < δ1 < δ2 such that

$$\delta_1|x|^p \leq V(x) \leq \delta_2|x|^p. \quad (8)$$

Then, system (1) is stable if these two equivalent statement is satisfied:

(i) the Lyapunov function is strictly negative for all k ∈ N with the increment of time sequence

$$\Delta V(k) = V\big(\chi_k(T_k)\big) - V\big(\chi_k(0)\big) < 0; \quad (9)$$

(ii) there exist a continuous and differentiable looped-function V, it satisfied definition 2.2 and

$$\mathrm{V}(0,\cdot) = \mathrm{V}(\mathrm{T},\cdot). \quad (10)$$

Proof. Assume that *(ii)* of theorem 2.1 is satisfied and τ ∈ [0, T_k], then

$$\int_0^{T_k}\dot{W}(\tau,\chi_k) = \int_0^{T_k}\frac{d}{d\tau}\big[V\big(\chi_k(\tau)\big) + V(\tau,\chi_k)\big]$$
$$= V\big(\chi_k(0)\big) + V(0,\chi_k) - V\big(\chi_k(T_k)\big) - V(T_k,\chi_k)$$
$$= V\big(\chi_k(0)\big) - V\big(\chi_k(T_k)\big)$$
$$=-\Big(V\big(\chi_{k+1}(0)\big) - V\big(\chi_k(0)\big)\Big). \quad (11)$$

Since the decreasing of $\{\,V\big(\chi_k(T_k)\big)\,\}$, so $V\big(\chi_{k+1}(0)\big)$ - $V\big(\chi_k(0)\big)$ < 0, therefore, the proof *(i)* is complete.

Assume that *(i)* of theorem 2.1 is satisfied. and we introduce a function

$$V(\tau,\chi_k,T_k) =-V\big(\chi_k(\tau)\big) + \frac{\tau}{T_k}\big[\Delta V\big(\chi_k(T_k)\big)\big]. \quad (12)$$

Obviously, it satisfied LF V(0,χ_k, T_k) = V(T_k, χ_k, T_k) .

Then, (7) can be wrote as following

$$\frac{d}{d\tau}W(\tau,\chi,\chi_{k-1}) = \frac{d}{d\tau}\big[V\big(\chi_k(\tau)\big) + V(\tau,\chi_k)\big]$$
$$= \dot{V}\big(\chi_k(\tau)\big) - \dot{V}\big(\chi_k(\tau)\big) + \frac{1}{T_k}\big[\Delta V\big(\chi_k(T_k)\big)\big]$$
$$= \frac{1}{T_k}\big[\Delta V\big(\chi_k(T_k)\big)\big] \quad (13)$$

From definition 2.3 and $\triangle V\big(\chi_k(T_k)\big) = V\big(\chi_k(T_k)\big)$-$V\big(\chi_k(0)\big)$< 0, so the system (1) is stable.

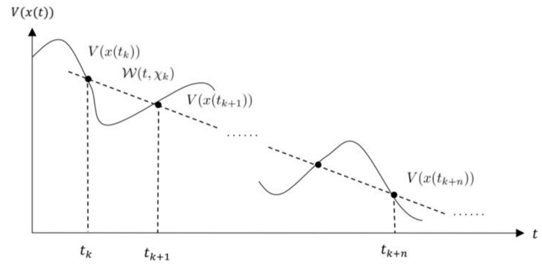

Figure 1. State of response of LF

In Figure 1, the real line denotes the continuous Lyapunov function, dashed denotes the discrete point of $W(\tau,\ \chi_k)$ = V(χ_k (τ)) + V(τ, χ_k) and the V(τ, χ_k) is LF. In this paper the important thing for the Lyapunov function is that the function is non-monotonic, we just need that it is monotonically decreasing at the discrete points and combine with the LF V(0, ·) = (V , ·). This method is the core theoretical method based on the discrete-time theorem in this article. In the Fig.1, we can clearly see that the Lyapunov function is non-monotonic, and we just need the sequence $\{t_k\}|$ k ∈ ℕ is decreasing, and at the same time in this instant the LF V(0, ·) = V(T, ·) = 0, so in this instant there exist $W(T_k\ ,\chi_k)$ = $V\big(\chi_k(T_k)\big)$, so the number of discrete points increases, eventually the system will be stable.

3. Impulsive Mechanism and Controller

In this section, first we introduce the sub-observer for impulsive switching system, then we introduce the triggered measure between sub-system and impulsive system. Finally analysis the system with the LF.

3.1 Sampling Observer

First, we introduce a sub-observer system of the impulsive system, then we analyst the impulsive sub-observer of LF.

$$\dot{\tilde{\chi}}_k(\tau) = A_\sigma\tilde{\chi}_k(\tau) + B_\sigma K_\sigma\tilde{\chi}_k(0) + L_\sigma(y_k - \tilde{y}_k)$$
(14)

Where $\tilde{\chi}_k(\tau)$ is the state of the kth sampling sub-system. $\tilde{y}_k$ is the output of sampling system. L_σ is observer gain of the subsystem. Let

$$e(t) = x(t) - \tilde{x}(t)e_k(\tau) = \chi_k(\tau) - \tilde{\chi}_k(\tau)$$

then

$$\frac{d}{d\tau}\big(e_k(\tau)\big)$$
$$= \frac{d}{d\tau}\big(x_k(\tau) - \tilde{x}_k(\tau)\big)$$
$$= A_\sigma\big(\chi_k(\tau) - \tilde{\chi}_k(\tau)\big) \quad (15)$$
$$- L_\sigma C_\sigma\big(\chi_k(\tau) - \tilde{\chi}_\chi(\tau)\big)$$

$= (A_\sigma - L_\sigma C_\sigma)e_k(\tau)$.

We can constructed from (14) as following

$$\dot{\widetilde{\chi}}_k(\tau) = A_\sigma\widetilde{\chi}_k(\tau) + B_\sigma K_\sigma\widetilde{\chi}_k(0) + L_\sigma C_\sigma\big(\chi_k(\tau) - \widetilde{\chi}_\chi(\tau)\big) \quad (16)$$
$$= A_\sigma\widetilde{\chi}_k(\tau) + B_\sigma K_\sigma\widetilde{\chi}_k(0) + L_\sigma C_\sigma e_k(\tau).$$

Consider (16) and let $V_\sigma(\tau) = e_k^T(\tau)Pe_k(\tau)$ as the quadratic Lyapunov function, the LF define as $V(\tau,\chi_k,T_k) =- V\big(\chi_k(\tau)\big) + \frac{\tau}{T_k}\big[\Delta V\big(\chi_k(T_k)\big)\big]$

From definition 2.1 have $\chi_k(\tau)$ and $\widetilde{\chi}_k(\tau)$ are approximately equal for the system not in the switching instant. Then we can get

$$\dot{W} = \frac{1}{T_k}\big[\Delta V\big(\chi_k(T_k)\big)\big]$$
$$= \frac{1}{T_k}\big[e_k^T(T_k)Pe_k(T_k) - e_k^T(0)Pe_k(0)\big]$$
$$= \frac{1}{T_k}\big[e_k^T(T_k)Pe_k(T_k) - e_{k-1}^T(T_k)Pe_{k-1}(T_k)\big]$$
$$= \frac{1}{T_k}\Big[\Big(\chi_k^T(T_k) - \widetilde{\chi}_k^T(T_k)\Big)P\big(\chi_k(T_k) - \widetilde{\chi}_k(T_k)\big) - \Big(\chi_{k-1}^T(T_k) - \widetilde{\chi}_{k-1}^T(T_k)\Big)P\big(\chi_{k-1}(T_k) - \widetilde{\chi}_{k-1}(T_k)\big)\Big]$$
$$= \frac{1}{T_k}\big[\chi_k^T(T_k)P\chi_k(T_k) + \widetilde{\chi}_k^T(T_k)P\widetilde{\chi}_k(T_k) - \chi_k^T(T_k)P\widetilde{\chi}_k(T_k) - \widetilde{\chi}_k^T(T_k)P\chi_k(T_k) - \chi_{k-1}^T(T_k)P\chi_{k-1}(T_k) - \widetilde{\chi}_{k-1}^T(T_k)P\widetilde{\chi}_{k-1}(T_k) + \chi_{k-1}^T(T_k)P\widetilde{\chi}_{k-1}(T_k) + \widetilde{\chi}_{k-1}^T(T_k)P\chi_{k-1}(T_k)\big]$$

Since

$$\chi_k^T(T_k)P\chi_k(T_k) - \chi_{k-1}^T(T_k)P\chi_{k-1}(T_k) < 0$$

then

$$\widetilde{\chi}_k^T(T_k)P\widetilde{\chi}_k(T_k) - \chi_k^T(T_k)P\widetilde{\chi}_k(T_k) - \widetilde{\chi}_k^T(T_k)P\chi_k(T_k) - \widetilde{\chi}_{k-1}^T(T_k)P\widetilde{\chi}_{k-1}(T_k) + \chi_{k-1}^T(T_k)P\widetilde{\chi}_{k-1}(T_k) + \widetilde{\chi}_{k-1}^T(T_k)P\chi_{k-1}(T_k) \le 0.$$

So $\dot{W} \le 0$, then the sampling observer mechanism is stable which implies that the system (1) is stable.

3.2 Sampled-Triggered Measure and Controller

In this sub-section, we introduce sampling measure and it is made up of two parts: sampling and monitoring. The monitor is extremely important for switching system, it determines generated or not of event. And the condition introduce by

$$|\widetilde{e}_k(\tau)|^2 \ge \mu|\widetilde{\chi}_k(\tau)|^2, \quad (17)$$

where $e(t) = x(t) - \hat{x}(t)e_k(\tau) = \chi_k(\tau) - \widetilde{\chi}_k(\tau)$ and

$\mu > 0$ is a constant threshold. The sampling will occur

immediately. When the event occurs, the error $e_k(\tau)$ is reset to zero and instants to grow until it happen again. In this paper we have the assumption, $\widetilde{t}_0 = t_0 = 0$ at the first event is occurred. From the above we can get

$$\widetilde{t}_{k+1} = inf\{t > \widetilde{t}_k \,|\, |\widetilde{e}_k(\tau)|^2 \ge \mu|\widetilde{\chi}_k(\tau)|^2\}, \quad (18)$$

Where $\widetilde{\tau}, \widetilde{t}_{k+1}$ are the state $\chi_k(0)$ sampled at time $\widetilde{t}_k$ and next instant.

4. Stability Analysis

In this paper switching between the two sampling intervals at most once. Suppose that $n = 1, 2, 3 \cdots$, samplings occur on the interval $[t_i, t_{i+1})$, from (14) we can get

$$\dot{\widetilde{\chi}}_k(\tau) = A_\sigma\widetilde{\chi}_k(\tau) + B_\sigma K_\sigma\widetilde{\chi}_k(\tau_k) + L_\sigma\big(y_{\chi_k} - \widetilde{y}_{\chi_k}\big)$$
$$= A_\sigma\widetilde{\chi}_k(\tau) + B_\sigma K_\sigma\widetilde{\chi}_k(\tau_k) + L_\sigma C_\sigma e_k(\tau)$$
$$= A_\sigma\widetilde{\chi}_k(\tau) + B_\sigma K_\sigma\big(\widetilde{\chi}_k(\tau) - \widetilde{e}_k(\tau)\big) + L_\sigma C_\sigma e_k(\tau) \quad (19)$$
$$= (A_\sigma + B_\sigma K_\sigma)\widetilde{\chi}_k(\tau) - B_\sigma K_\sigma\widetilde{e}_k(\tau) + L_\sigma C_\sigma e_k(\tau)$$

From (15) and (19), we can get

$$\begin{cases}\dot{\widetilde{\chi}}_k(\tau) = (A_\sigma + B_\sigma K_\sigma)\widetilde{\chi}_k(\tau) - B_\sigma K_\sigma\widetilde{e}_k(\tau) + L_\sigma C_\sigma e_k(\tau) \\ \dot{e}_k(\tau) = (A_\sigma - L_\sigma C_\sigma)e_k(\tau)\end{cases} \quad (20)$$

and we can rewritten it as

$$\dot{\zeta}(\tau) = \bar{A}_\sigma\zeta(\tau) + \bar{B}_\sigma\hat{e}(\tau) \quad (21)$$

where

$$\zeta(\tau) = \begin{bmatrix}\widetilde{\chi}_k(\tau)\\ e_k(\tau)\end{bmatrix}, \bar{A}_\sigma = \begin{bmatrix}A_\sigma + B_\sigma K_\sigma & L_\sigma C_\sigma\\ 0 & A_\sigma - L_\sigma C_\sigma\end{bmatrix}$$
$$\bar{B}_\sigma = \begin{bmatrix}-B_\sigma K_\sigma & 0\\ 0 & 0\end{bmatrix}, \hat{e}(\tau) = \begin{bmatrix}\widetilde{e}_{\chi_k}(\tau)\\ 0\end{bmatrix}$$

Lemma 4.1. There exist matrix $R > 0$, $N > 0$ and $\tau \in [0, T_k]$, the following is satisfied

$$-\int_0^\tau \dot{\chi}_k^T(\theta)R\dot{\chi}_k(\theta)d\theta \le 2F_k^T(\tau)N\big(\chi_k(\tau) - \chi_k(0)\big) - \tau F_k^T(\tau)NR^{-1}N^TF_k(\tau)$$

where $F_k = [\chi_k(\tau) \;\; \chi_k(0)]^T$.

Theorem 4.1. There exist $P > 0$, $R > 0$, $C_1 \in S^n$, $X \in S^n$, $C2 \in \mathbb{R}^{n\times n,}$ and $N \in \mathbb{R}^{2n\times n}$. If those *LMIs* are satisfied for all $\tau \in [0, T_k]$

$$\Pi_1 + \Pi_2 + \Pi_3 + \Pi_3 + \Pi_5 + \Pi_6 < 0,$$

where

$$M = [A_\sigma B_\sigma K_\sigma],$$
$$e_1 = [1\ 0]^T,$$
$$e_2 = [0\ 1]^T, e = [e_1\ e_2], e_{12} = e_1 - e_2,$$
$$\pi = T_k - \tau,$$
$$F_k = [\chi_k(\tau)\chi_k(0)]^T,$$
$$\Pi_1 = - e_{12}C_1 e_{12}^T + 2\pi e_{12}C_1 Me,$$
$$\Pi_2 = - 2e_{12}C_2 e_2 + 2\pi MeC_2 e_2,$$
$$\Pi_3 = 2Ne_1^T - 2Ne_2^T - \pi NR^{-1}N + \pi MeR(Me)^T,$$
$$\Pi_4 = e_2 T_k e_2^T - 2e_2\tau e_2^T,$$
$$\Pi_5 = 2e_1 PMe$$

Then the system (21) is asymptotically stable for sampling system satisfying system (1).

Proof. We choose the Lyapunov function as following:

$$W(\tau, \chi_k) = V(\chi_k(\tau)) + \textstyle\sum_1^4 V_i(\tau, \chi_k),$$

where $\tau \in [0, T_k]$ and let $\pi = T_k - \tau$, then the components are given by

$$V(\chi_k(\tau)) = \chi_k(\tau)^T P \chi_k(\tau),$$
$$V(\tau, \chi_k) = \textstyle\sum_1^4 V_i(\tau, \chi_k),$$
$$V_1(\tau, \chi_k) = \pi\zeta_k^T(\tau)C_1\zeta_k(\tau),$$
$$V_2(\tau, \chi_k) = 2\pi\zeta_k^T(\tau)C_2\chi_k(0),$$
$$V_3(\tau, \chi_k) = \pi\int_0^\tau \dot{\chi}_k^T(\theta)R\dot{\chi}_k(\theta)d\theta,$$
$$V_4(\tau, \chi_k) = \pi\tau\chi_k^T(0)X\chi_k(0).$$

So we can get

$$\dot{W}(\tau, \chi_k) = \dot{V}(\chi_k(\tau)) + \textstyle\sum_1^4 \dot{V}_i(\tau, \chi_k), \quad (22)$$

where

$$\dot{V}_1(\tau, \chi_k) = F_k^T \Pi_1 F_k,$$
$$\dot{V}_2(\tau, \chi_k) = F_k^T \Pi_2 F_k,$$
$$\dot{V}_3(\tau, \chi_k) \le F_k^T \Pi_3 F_k,$$
$$\dot{V}_4(\tau, \chi_k) = F_k^T \Pi_4 F_k,$$
$$\dot{V}(\chi_k(\tau)) = F_k^T \Pi_5 F_k.$$

And from lemma 4.1we can get

$$\dot{V}_3(\tau, \chi_k) \le F_k^T \Pi_3 F_k \quad (23)$$

So $\dot{W}(\tau, \chi_k) < 0$, then the system (1) is stable. This proof is complete.

4. Conclusion

This paper used a new LF to solve the stability of linear switching systems with sampled-triggered control. The method is based on the discrete-time Lyapunov theorem. It greatly reduces the conservation of the system. Firstly, the system is divided into each corresponding interval to analyze by solving the differential equation of the system. Then the stability of the system is analyzed by introducing the LF. Based on the LF, it can improve criteria of the stability. The combination of LF and switching system can greatly reduce the amount of calculation and improve the stability of switching system.

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