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Simulation of Time-Independent Aharanov-Bohm Effect Using Matlab

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Abstract: We simulated time-varying Aharanov-Bohm effect (ABE) under time-independent magnetic field using Matlab, taking into account the influence of time-varying magnetic field on charged particles moving in a two-dimensional plane. According to our models, we discovered the time evolution of wave function and interference patterns. We also developed two mathematical models to analyze this phenomenon, of which the first mode focuses on the time evolution of the wave function, governed by the time-dependent Schrödinger equation incorporating electromagnetic potentials, while the second one is responsible for simulating the resulting interference patterns by modifying the intensity of wave packet based on the frequency-dependent phase shift caused by the sinusoidal magnetic field in the solenoid. Our results align with theoretical expectations and provide new insights into the frequency-dependent nature of the time-varying ABE.

Keywords: Aharanov-Bohm Effect (ABE); Time-Dependent Schrödinger Equation; Wave Function Evolution; Interference Patterns

1. Introduction

The Aharanov-Bohm effect (ABE) for steady magnetic fields is well established both theoretically and experimentally [1]. It is a quantum phenomenon typically observed in a two-slit interference experiment, involving charged particles, such as electrons, as they traverse paths on either side of a solenoid carrying a current. Under ideal conditions, that is, infinitely long solenoid with a constant current, electrons pass through regions where electric and magnetic fields are absent. Nevertheless, the interference pattern is altered due to the influence of the vector potential generated by the current of the solenoid. This phenomenon is commonly referred to in the literature as Type I Aharonov-Bohm effect (ABE I). In contrast, when the current of the solenoid varies over time, time-dependent electromagnetic conditions will be created, resulting in the presence of non-zero electric and magnetic fields along the trajectory of electrons. In this case, the changes in the interference pattern arise from both the vector potential and the accompanying fields. This variation is identified as Type II Aharonov-Bohm effect (ABE II) which was discussed by Batelaan and Tonomura [2]. A typical example of a Type II effect is the Aharonov–Casher effect [3], where a neutral particle with a magnetic moment moves through an electric field and, by doing so, picks up a Aharonov-Bohm like phase. This phenomenon highlights the significance of electromagnetic potentials in the quantum theory, emphasizing that a charged particle can experience a phase shift in the absence of any classical electromagnetic force. Specifically, the AB effect occurs when an electron beam encircles a region with a confined magnetic field, resulting in an observable phase difference, even though electrons travel through a field-free region. This counterintuitive result underscores the foundational role of potential in quantum mechanics and challenges classical intuitions about the nature of forces and fields.

There is no clear consensus in existing literature on whether Type II Aharonov-Bohm effect (ABE II) leads to a non-zero shift in the interference pattern. One old experiment [4] suggested that the effects of the electric field, magnetic field, and vector potential might counterbalance each other, leaving no fringe shift. This uncertainty has been succinctly highlighted by Jian Jing [5].

Choudhury and his fellow researchers [6] conducted several experiments and published a simpler version calculation to facilitate the understanding of time-varying ABE by a direct calculation. They first created a simple graph of Aharonov-Bohm effect, as shown in Figure 1[6], on the basis of which they performed calculations.

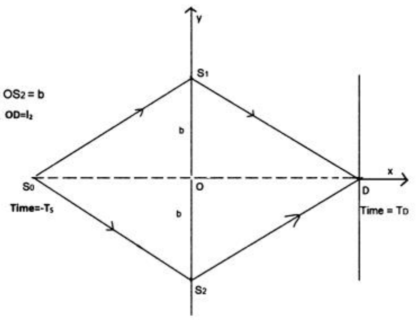

Figure 1. Aharonov-Bohm Effect

They revealed a frequency-dependent AB phase shift, which converged to a constant magnetic field at low frequencies. However, at high frequencies, rapid oscillations in the AB phase led to a negligible net effect. Choudhury highlights that while the electric field from time-varying magnetic field does not directly contribute to the action integral despite its impact on the trajectory of electrons. Their analysis shows that higher-order terms in the vector potential are negligible for typical field strengths, ensuring the robustness of their results.

Based on previous research about Aharonov-Bohm effect, we constructed two mathematical models in order to analyze the ABE through Matlab. Our experimental results reveal unique insights into the time-dependent Aharonov-Bohm effect, offering observations that differ from those of previous studies. Compared to earlier research, our findings highlight distinct differences in the behavior of interference pattern under varying conditions, providing a novel perspective on the interaction between charged particles and time-dependent vector potential. Moreover, this study serves as a significant complement to existing research on the Aharonov-Bohm effect. By addressing previously unexplored aspects of the time-dependent ABE, our work not only reinforces but also extends the theoretical and experimental understanding of electromagnetic potential in quantum mechanics.

2. Methodologies

We mainly employed two models, one is named "Wave Function Evolution", as illustrated in Figure 2.

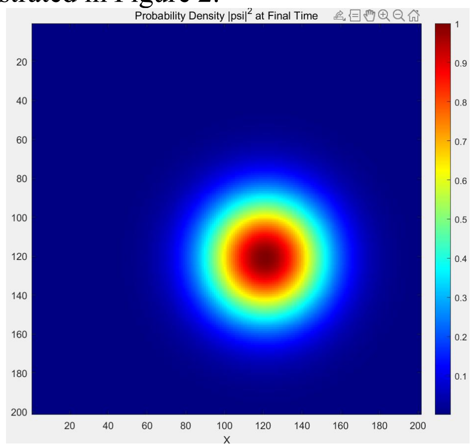

Figure 2. Wave Function Evolution

The time-dependent Schrödinger equation with an electromagnetic field describes how the wave function of a quantum particle evolves over time in the presence of electric fields. This is crucial for understanding phenomena in quantum mechanics where particles interact with external fields, such as in atomic and molecular physics as well as solid-state physics.

The inclusion of the electromagnetic vector potential A(x, y, t) allows the equation to model the influence of electric and magnetic fields on the quantum state, thus providing a comprehensive framework for understanding quantum dynamics in the presence of external fields. the phase accumulated by the electron wavefunction in various paths was computed using the path integral method. the time-dependent magnetic vector potential A(t) was incorporated into the action integral, which directly influenced the AB phase. the electric field arising from the time-varying magnetic field was excluded from the action integral, as it primarily alters the classical path of the electron without contributing directly to the quantum phase shift.

In quantum mechanics, the wave function ψ(x, y, t) represents the quantum state of a particle. the square of the magnitude of the wave function ｜ψ(x, y, t) |^2 gives the probability density of finding the particle at a position (x, y) at time t.

The time-dependent Schrödinger equation in the presence of an electromagnetic field is expressed as:

$$i\hbar\frac{\partial\psi(x,y,t)}{\partial t} = \left[\frac{1}{2m}\left(-i\hbar\nabla - q\mathbf{A}(x,y,t)\right)^2 + V(x,y)\right]\psi(x,y,t) \quad (1)$$

This equation describes the quantum dynamics of a charged particle under the influence of electromagnetic fields, taking into account both the vector potential A (x, y, t) and the scalar potential ϕ (x, y, t).

For the second model (shown in Figure 3), we simulated an interference pattern by multiplying the intensity of the initial wave packet by a factor dependent on the phase shift, thereby visualizing how this pattern might evolve over time in the presence of a time-varying magnetic field.

The central bright spot corresponds to the region of maximum intensity, where the wave packet constructively interferes. Surrounding the central region, the intensity rapidly decreases, indicated by the transition from yellow to blue. the circular symmetry of the pattern suggests the wave packet is radially symmetric, likely due to uniform conditions during the simulation.

Phase shift is likely to vary over time due to the magnetic field. In quantum mechanics, this could relate to the Aharonov-Bohm effect, where the phase of a charged particle is affected by a magnetic vector potential even in regions where the magnetic field is zero.

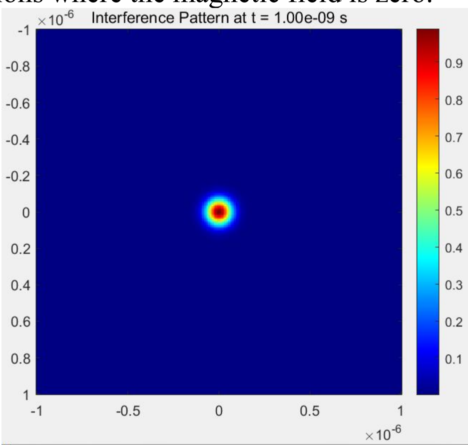

Figure 3. Second Model

The models constructed in this study offer an illustrative approach to understanding how a time-varying magnetic field might affect the interference pattern of the wave function of a charged particle.

3. Discussion and Conclusion

In this study, we have presented a comprehensive simulation of the time-varying Aharonov-Bohm effect utilizing Matlab. Through meticulous modeling and numerical analysis, we have explored the dynamical behavior of a charged particle in a two-dimensional plane subjected to a weak, time-varying magnetic field.

The simulation results confirm that the phase shift of the wave function varies with frequency in the presence of time-dependent magnetic vector potential. At low frequencies, the system's behavior converges to the classical ABE scenario for a static magnetic field because the magnetic field remains approximately constant in the transit process of electrons, resulting in a predictable phase shift and a stable interference pattern. Conversely, at higher frequencies, the rapid oscillation of the magnetic field leads to a more complex phase evolution, thus introducing fluctuations in the interference pattern, as evidenced by the blurring or diminishing intensity of the fringes in the high-frequency regime. This finding aligns with theoretical expectations, reinforcing the significance of the phase coherence time in ABE II. Our analysis underscores the impact of the time-dependent electric field on the classical trajectories of electrons. While this field does not directly contribute to the phase integral, it modifies the paths of electrons, influencing the spatial configuration of the interference pattern. the observed shift and distortion in the interference fringes provide a quantitative measure of the interplay between the time-dependent magnetic field and the evolution of the wave function.

Our findings reveal significant insights into the time evolution of the wave function and the resultant interference patterns, underscoring the profound implications of electromagnetic potentials in quantum mechanics. These findings contribute to a deeper understanding of the dynamic aspects of the Aharonov-Bohm effect and pave ways for future experimental validation. Potential applications of this research extend to quantum systems where time-dependent fields are prevalent, such as in quantum computing and electromagnetic wave-driven quantum systems.

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